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Author(s): Pierluigi Balduzzi, Sanjiv Ranjan Das, Silverio Foresi

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THE CENTRAL TENDENCY: A SECOND FACTOR IN BOND YIELDS

Pierluigi Balduzzi, Sanjiv Ranjan Das, and Silverio Foresi*

Abstract—We assume that the instantaneous riskless rate reverts toward a central tendency which, in turn, is changing stochastically over time. As a result, current short-term rates are not sufficient to predict future short-term rate movements, as it would be the case if the central tendency were constant. However, since longer maturity bond prices incorporate information about the central tendency, longer maturity bond yields can be used to predict future short-term rate movements. We develop a two-factor model of the term structure which implies that a linear combination of any two rates can be used as a proxy for the central tendency. Based on this central-tendency proxy, we estimate a model of the one-month rate that performs better than models which assume the central tendency to be constant.

I. Introduction

THIS PAPER develops a two-factor model of the term structure of interest rates. We follow the standard tradition in the finance term-structure literature (see, for example, Brennan and Schwartz (1979), Cox et al. (1981, 1985), Longstaff and Schwartz (1992), and Vasicek (1977)), and identify the first factor with the *level* of the short-term rate. The novelty of our analysis is that we identify the second factor with the *central tendency* of the short-term rate. Our model captures the notion that short-term rates display short-lived fluctuations around a time-varying rest level, the central tendency, which, in turn, changes stochastically over time. As a result, current short-term rates are not sufficient to predict future short-term rate movements, as would be the case if the central tendency were constant over time. However, since longer maturity bond prices incorporate information about the central tendency, longer maturity bond yields can be used to predict future short-term rate movements.

Our study integrates two strands of the literature. The first strand of the literature is the traditional regression-based work on the term structure of interest rates, which uses information in longer term rates to forecast future short-term rates. Examples of this approach are Campbell and Shiller (1992), Fama (1984), Fama and Bliss (1987), Mankiw (1986), Shiller (1979), and Shiller et al. (1983). The second strand of literature uses no-arbitrage arguments to derive implications for longer term yields based on the dynamics of the short-term rate. Classical examples of this second approach include Cox et al. (1985) and Vasicek (1977). We take a middle course between the two approaches in the sense that we show how an equilibrium term-structure model can be used to learn about the dynamics of the short-term rate.

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* New York University, Harvard University, and New York University, respectively.

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Our analysis gathers some preliminary evidence on the explanatory power of longer-term yields for the short-term rate. We regress the next-period one-month rate on the current one-month rate and a longer maturity bond yield; we find that the bond yield always has explanatory power, especially for the shorter maturities. This evidence motivates a two-factor term-structure model, which is based on the assumption that the instantaneous riskless rate reverts toward a central tendency, which, in turn, changes stochastically over time. We impose minimal restrictions on the dynamics of the central tendency, other than that its conditional mean should not be affected by the instantaneous riskless rate. The theoretical analysis suggests a proxy for the central-tendency factor which depends on a linear combination of two yields. The central-tendency proxy is then used to estimate the stochastic process for the instantaneous riskless rate, proxied by the one-month Treasury bill rate.

In the empirical analysis we show that models of the one-month rate which allow for a time-varying central tendency perform better than the standard Vasicek (1977) and Cox et al. (1985) models, in which the central tendency is assumed constant. We replicate the analysis for different subperiods. The time variation in the central tendency is especially important for the 1952–1971 and 1982–1993 periods, when the Federal Reserve's operating policy took into account both the volatility of interest rates and the behavior of monetary aggregates.

The approach advocated here complements other two-factor models of the term structure of interest rates such as that of Longstaff and Schwartz (1992), who propose a model in which the second factor is identified with the conditional volatility of the short-term rate. Also, in a spirit similar to ours, Babbs and Webber (1995), Naik and Lee (1994), and Pennacchi and Jagadeesh (1995) postulate a process for the short-term rate where its central tendency changes over time.

The paper is organized as follows. Section II presents the data and puts forward the motivating evidence, while section III develops the theoretical model. In section IV we estimate various models of the one-month rate and we examine the properties of the central-tendency proxy. Section V performs diagnostics of the interest rate model, and section VI concludes.

II. Preliminary Evidence

This section illustrates the properties of the data, and puts forward some preliminary evidence that longer maturity yields contain information for future short-term interest rates.

Table 1 presents summary statistics for the interest rate series used in the empirical analysis. We use monthly U.S. interest rates from June 1952 through December 1993. One

TABLE 1.—SUMMARY STATISTICS OF SELECTED U.S. INTEREST RATES
A. LEVELS

Maturity	Sample Mean	Standard Deviation	Skewness ^a	Kurtosis ^a	First-Order Autocorrelation
<i>June 1952–December 1993</i>					
1 month	5.113	2.936	0.983	1.059	0.966
1 year	5.954	3.057	0.745	0.351	0.982
2 years	6.138	3.022	0.685	0.135	0.986
3 years	6.293	2.968	0.645	0.040	0.987
4 years	6.408	2.951	0.606	-0.051	0.988
5 years	6.479	2.923	0.584	-0.160	0.989
<i>June 1952–August 1971</i>					
1 month	3.153	1.566	0.551	-0.298	0.967
1 year	3.788	1.675	0.446	-0.267	0.977
2 years	3.864	1.576	0.534	-0.056	0.976
3 years	3.997	1.531	0.566	-0.067	0.977
4 years	4.069	1.497	0.591	-0.006	0.976
5 years	4.121	1.446	0.650	0.036	0.976
<i>September 1971–September 1979</i>					
1 month	6.110	1.836	0.489	-0.721	0.917
1 year	6.909	1.588	0.462	-0.551	0.912
2 years	7.043	1.281	0.367	-0.462	0.917
3 years	7.118	1.084	0.268	-0.560	0.909
4 years	7.199	0.997	0.152	-0.632	0.906
5 years	7.257	0.928	0.087	-0.720	0.917
<i>October 1979–September 1982</i>					
1 month	11.545	2.647	-0.419	-0.640	0.708
1 year	12.671	1.909	-0.651	-0.045	0.686
2 years	12.593	1.716	-0.487	-0.038	0.751
3 years	12.493	1.657	-0.419	-0.349	0.795
4 years	12.401	1.650	-0.301	-0.524	0.817
5 years	12.341	1.514	-0.391	-0.741	0.826
<i>October 1982–December 1993</i>					
1 month	6.043	2.058	-0.158	-0.886	0.919
1 year	7.190	2.168	-0.193	-0.549	0.969
2 years	7.664	2.153	0.015	-0.454	0.970
3 years	7.980	2.081	0.123	-0.427	0.966
4 years	8.248	2.025	0.237	-0.413	0.967
5 years	8.397	1.986	0.344	-0.360	0.967
B. FIRST DIFFERENCES					
Maturity	Sample Mean	Standard Deviation	Skewness ^a	Kurtosis ^a	First-Order Autocorrelation
<i>July 1952–December 1993</i>					
1 month	0.002	0.735	-0.863	10.072	-0.089
1 year	0.003	0.538	-1.083	13.411	0.116
2 years	0.004	0.462	-0.724	10.161	0.154
3 years	0.005	0.421	-0.177	7.562	0.109
4 years	0.006	0.405	-0.210	5.323	0.060
5 years	0.006	0.375	-0.330	5.031	0.071
<i>July 1952–August 1971</i>					
1 month	0.011	0.378	-0.009	1.925	-0.200
1 year	0.014	0.319	-0.042	3.153	0.163
2 years	0.015	0.299	0.022	4.526	0.090
3 years	0.015	0.279	-0.304	4.620	0.033
4 years	0.016	0.268	-0.197	4.017	-0.003
5 years	0.017	0.253	-0.165	4.026	0.014
<i>September 1971–September 1979</i>					
1 month	0.059	0.583	-1.614	8.372	-0.137
1 year	0.056	0.486	-0.572	1.670	-0.007
2 years	0.046	0.379	-0.068	0.288	0.016
3 years	0.040	0.338	0.087	0.594	-0.026
4 years	0.036	0.332	0.244	1.355	-0.075
5 years	0.032	0.282	0.023	0.692	-0.068
<i>October 1979–September 1982</i>					
1 month	-0.085	1.883	-0.535	0.592	0.036
1 year	-0.012	1.482	-0.701	1.126	0.101
2 years	0.031	1.207	-0.676	1.034	0.163
3 years	0.045	1.050	-0.283	0.653	0.127
4 years	0.053	0.972	-0.404	0.122	0.054
5 years	0.064	0.866	-0.522	0.441	0.072
<i>November 1982–December 1993</i>					
1 month	-0.030	0.764	0.213	2.857	-0.245
1 year	-0.048	0.393	0.103	0.418	0.134
2 years	-0.050	0.388	0.130	0.019	0.209
3 years	-0.049	0.386	0.306	-0.012	0.138
4 years	-0.045	0.396	0.309	0.145	0.099
5 years	-0.045	0.394	0.100	0.136	0.071

Note: ^a Standardized measures of third and fourth moment, zero for normal distribution.

month Treasury bill rates are from the Center for Research in Security Prices (CRSP) RISKFREE file, while the discount-bond prices used to compute continuously compounded yields are from the FAMABLIS file also on the CRSP tape. All interest rates are continuously compounded. We consider four separate subperiods, June 1952–August 1971, September 1971–September 1979, October 1979–September 1982, and October 1982–December 1993.

The subperiods were selected to control for different operating procedures followed by the Federal Reserve. During the 1950s and 1960s the Federal Reserve took the view that monetary policy should be based on the intuitive judgment of money-market conditions. This period also roughly coincides with the time during which the Bretton Woods exchange-rate agreement was in place, which ended in August 1971. During the 1970s the Federal Reserve made a formal commitment to targeting the growth rate of key monetary aggregates, such as M1 and M2. In practice, this translated into a tight targeting of the federal funds rate at a level perceived to be consistent with the stated quantity targets. The October 1979–October 1982 period witnessed the Federal Reserve's "experiment." The operating target of monetary policy became the amount of nonborrowed reserves with the banking system, and both the level and the volatility of interest rates reached levels never experienced before. The post-October 1982 period is one of de-emphasis of monetary aggregates and of renewed concern for reducing the volatility of interest rates.

While it is the changing institutional setting of postwar monetary policy that suggests the years 1971, 1979, and 1982 as possible break points, the evidence in table 1 confirms that interest rates behaved quite differently across subperiods. Mean interest rates are highest in the third subperiod and lowest in the first one; the volatility of the changes of all rates is also highest in the third subperiod and lowest in the first one. The skewness of the levels of interest rates is highest (and positive) in the first subperiod, while it is lowest (and negative) in the third subperiod. This behavior is consistent with the notion that interest rates cannot turn negative, and that when interest rates are especially high, large sudden drops are quite likely. Changes in interest rates also display marked excess kurtosis over the entire period, especially for the shorter maturity rates, while this excess kurtosis disappears during the individual subperiods. This is consistent with the view that controlling for variations in means and variances across subperiods, as in Naik and Lee (1994) and in this paper, may improve the performance of models with Gaussian innovations. Table 1 also shows that all rates exhibit high first-order autocorrelations for the first subperiod; and the higher order autocorrelations (not shown in the table) decay very slowly. Fama (1984, p. 515) writes that this "pattern [is] suggestive of mean non-stationarity," as it would be the case with breaks in regimes and changes in the central tendency. In fact, the first-order autocorrelations for the individual subperiods are lower than for the entire period. Given the persistence of the levels of all rates, when

appropriate we conduct our analysis both in levels and in first differences. This approach is consistent, for example, with the findings of Campbell and Shiller (1987), who provide strong evidence of unit roots in the monthly series of U.S. interest rates for the 1959–1978 period.

With a time-varying central tendency, current short-term rates do not adequately summarize the information in the bond market about future short-term rates. Hence, in addition to the current short-term rate, we should use longer maturity yields. To explore this idea we regress the future short-term rate on the current short-term rate and current longer term yields,

$$r_{t+1} = \beta_0 + \beta_1 r_t + \beta_2 y_t(\tau) + \text{error}_{t+1} \quad (1)$$

where $y(\tau)$ is the yield on a discount bond of maturity $\tau = 1, 2, 3, 4,$ and 5 years, and r is the one-month interest rate. This regression is performed for both the levels and the first differences of the interest rates. If interest rates were driven by a single factor following an autoregressive process as in Cox et al. (1985) or Vasicek (1977), current short-term rates would completely summarize the information about future short-term rates, and we should find that $\beta_2 = 0$.

Table 2 reports the results of estimating the statistical model (1), both in levels and in first differences, for the entire period and for the different subperiods. The main finding is that for the entire period the estimates of all coefficients are significant. In particular β_2 , the coefficient of the long-term yield, is significant for the regression in both levels and first differences. This finding confirms that there is information in the longer maturity yields that can be used to explain future short-term rate variations. Longer term rates are more significant in the first and fourth subperiods. Thus we expect a model with a time-varying central tendency to perform better in these subperiods than in the second and third subperiods. Also, the coefficient of the long-term yield β_2 tends to decrease with the maturity of the yield used in the regression, especially for the regression in levels. This finding is consistent with the notion that the shorter the maturity of the bond yield, the higher the information on proximate short-term rate changes. When we consider the regression in the levels, we also find that the coefficient of the short-term rate β_1 tends to increase with the maturity. In fact, in all regressions the sum of β_1 and β_2 is approximately equal to 1. This is consistent with the evidence that interest rates of all maturities are very persistent, as is the case with integrated series, and with the notion that interest rates may be cointegrated. In fact, Campbell and Shiller (1987) provide evidence of a cointegrating relationship between short-term and long-term rates, as implied by the expectations hypothesis.

III. Yields with a Time-Varying Central Tendency

The evidence presented in the previous section suggests that longer term bond yields help forecast future short-term rates, even when we control for information contained in the

TABLE 2.—LONG-TERM YIELD INFORMATION ON FUTURE SHORT-TERM RATES
A. LEVELS

Maturity	β_1	β_2	Adjusted R^2	Durbin-Watson Statistic
<i>July 1952–December 1993</i>				
1 year	0.624 (0.080)	0.338 (0.075)	0.944	1.849
2 years	0.763 (0.065)	0.207 (0.059)	0.941	1.925
3 years	0.823 (0.059)	0.152 (0.052)	0.940	1.980
4 years	0.847 (0.054)	0.129 (0.047)	0.940	1.996
5 years	0.865 (0.051)	0.111 (0.044)	0.939	2.019
<i>July 1952–August 1971</i>				
1 year	0.662 (0.073)	0.297 (0.068)	0.948	2.088
2 years	0.733 (0.066)	0.245 (0.066)	0.946	2.156
3 years	0.788 (0.054)	0.196 (0.056)	0.945	2.198
4 years	0.830 (0.050)	0.156 (0.054)	0.944	2.225
5 years	0.827 (0.051)	0.165 (0.057)	0.944	2.206
<i>September 1971–September 1979</i>				
1 year	0.721 (0.123)	0.317 (0.143)	0.904	2.080
2 years	0.873 (0.094)	0.179 (0.113)	0.902	2.084
3 years	0.904 (0.077)	0.161 (0.109)	0.902	2.101
4 years	0.931 (0.072)	0.123 (0.108)	0.901	2.116
5 years	0.949 (0.062)	0.093 (0.098)	0.900	2.128
<i>October 1979–September 1982</i>				
1 year	0.726 (0.226)	0.083 (0.313)	0.549	1.623
2 years	0.771 (0.188)	0.015 (0.280)	0.548	1.657
3 years	0.792 (0.171)	−0.028 (0.255)	0.548	1.681
4 years	0.756 (0.167)	0.055 (0.250)	0.549	1.634
5 years	0.785 (0.162)	−0.017 (0.273)	0.548	1.674
<i>October 1982–December 1993</i>				
1 year	0.447 (0.110)	0.499 (0.099)	0.900	2.023
2 years	0.554 (0.099)	0.400 (0.090)	0.893	2.067
3 years	0.626 (0.089)	0.342 (0.084)	0.887	2.123
4 years	0.684 (0.079)	0.291 (0.077)	0.882	2.154
5 years	0.711 (0.075)	0.268 (0.074)	0.880	2.188

TABLE 2.—(CONTINUED)
B. FIRST DIFFERENCES

Maturity	β_1	β_2	Adjusted R^2	Durbin-Watson Statistic
1 year	-0.356 (0.081)	0.607 (0.104)	0.131	1.951
2 years	-0.301 (0.077)	0.645 (0.113)	0.124	1.952
3 years	-0.272 (0.083)	0.645 (0.142)	0.107	1.949
4 years	-0.237 (0.081)	0.641 (0.139)	0.107	1.962
5 years	-0.223 (0.080)	0.616 (0.142)	0.085	1.945
<i>August 1952–August 1971</i>				
1 year	-0.401 (0.083)	0.479 (0.096)	0.156	2.153
2 years	-0.355 (0.087)	0.406 (0.102)	0.111	2.127
3 years	-0.345 (0.082)	0.417 (0.098)	0.107	2.135
4 years	-0.328 (0.084)	0.398 (0.104)	0.095	2.117
5 years	-0.317 (0.084)	0.438 (0.115)	0.105	2.112
<i>September 1971–September 1979</i>				
1 year	-0.348 (0.188)	0.359 (0.155)	0.036	2.006
2 years	-0.263 (0.192)	0.376 (0.175)	0.036	1.983
3 years	-0.270 (0.191)	0.497 (0.213)	0.056	1.980
4 years	-0.233 (0.189)	0.424 (0.220)	0.041	1.968
5 years	-0.249 (0.182)	0.553 (0.232)	0.050	1.975
<i>October 1979–September 1982</i>				
1 year	-0.441 (0.212)	0.809 (0.242)	0.127	1.603
2 years	-0.373 (0.180)	0.930 (0.250)	0.137	1.567
3 years	-0.327 (0.212)	0.974 (0.386)	0.112	1.547
4 years	-0.258 (0.182)	1.006 (0.362)	0.134	1.567
5 years	-0.243 (0.179)	0.990 (0.401)	0.076	1.564
<i>October 1982–December 1993</i>				
1 year	-0.315 (0.097)	0.557 (0.197)	0.125	2.098
2 years	-0.305 (0.095)	0.554 (0.199)	0.123	2.086
3 years	-0.295 (0.098)	0.472 (0.211)	0.102	2.082
4 years	-0.283 (0.096)	0.458 (0.201)	0.103	2.082
5 years	-0.279 (0.100)	0.398 (0.205)	0.088	2.075

Note: Standard errors, adjusted for heteroskedasticity (White (1980)), are in parentheses.

current level of the short-term rate. Hence in this section we develop an equilibrium model of the term structure of interest rates, in which longer term yields depend on the time-varying central tendency of the short-term rate. The resulting theory suggests that an appropriate combination of bond yields proxies for the central tendency of the postulated short-term rate process.

The behavior of the instantaneous riskless rate r is described by the stochastic differential equation¹

$$dr = k(\theta - r)dt + \sqrt{\sigma_0^2 + \sigma_1^2 r} dZ \quad (2)$$

where k , σ_0 , and σ_1 are constants, and Z is a standard Brownian motion; θ is the central tendency toward which the instantaneous rate reverts. As in Pearson and Sun (1994), the diffusion term of equation (2) generalizes the square-root process of Cox et al. (1985) by allowing the lower bound for the instantaneous interest rate to be different from zero.

In turn, θ evolves over time according to the stochastic differential equation

$$d\theta = m(\theta) dt + s(\theta) dW \quad (3)$$

where, like Z , W is a standard Brownian motion. We assume that the conditional drift $m(\theta)$ and the conditional volatility $s(\theta)$ are “smooth” (continuous with continuous derivatives) functions of θ alone. Specifically, θ may evolve according to

$$d\theta = (m_0 + m_1\theta) dt + \sqrt{s_0^2 + s_1^2\theta} dW. \quad (3')$$

The covariance between the two factors, $dr d\theta/dt = \sigma_{r\theta}$, is assumed constant.

We assume the premium required to compensate investors for the risk of fluctuations of r to be linear in r , $\lambda_0 + \lambda_1 r$, with λ_0 and λ_1 constant, while the premium required to compensate investors for the risk of fluctuations of θ , $l(\theta)$, is a smooth function of θ alone. Under the above assumptions the price of a risk-free discount bond of maturity τ , $P = P(r, \theta; \tau)$, satisfies the partial differential equation (see Cox et al. (1985))

$$E(\mathcal{D}P) - rP - P_r(\lambda_0 + \lambda_1 r) - P_\theta l(\theta) = 0 \quad (4)$$

where \mathcal{D} denotes the Dynkin differential operator.²

¹ We use the same notation to indicate the instantaneous riskless rate and the one-month rate, and in the empirical applications of section IV we use the one-month rate to proxy for the instantaneous riskless rate.

² Equation (4) can be written in the more intuitive excess-returns form

$$\frac{E(\mathcal{D}P)}{P} - r = \frac{P_r}{P}(\lambda_0 + \lambda_1 r) + \frac{P_\theta}{P}l(\theta)$$

where P_r/P can be interpreted as the short-term rate beta, and P_θ/P as the central-tendency beta. By analogy with standard linear pricing models, $(\lambda_0 + \lambda_1 r)$ and $l(\theta)$ are the factor risk premiums associated with r and θ , respectively.

We assume that there is a solution for the fundamental valuation equation (4). The solution is of the form

$$P(r, \theta; \tau) = e^{-A(\theta; \tau) - B(\tau)r} \quad (5)$$

where $B(\tau)$ is a constant that depends only on the maturity τ , while $A(\theta; \tau)$ is a smooth function of θ , for any given maturity τ . In fact one can write equation (4) as

$$\begin{aligned} B(\tau)[k\theta - \lambda_0 - (k + \lambda_1)r] + B^2(\tau) \left[\frac{\sigma_0^2 + \sigma_1^2 r + \sigma_2^2 \theta}{2} \right] \\ + A_\theta(\theta; \tau)[m(\theta) - l(\theta)] + [A_{\theta\theta}(\theta; \tau) + A_\theta^2(\theta; \tau)] \frac{s(\theta)^2}{2} \\ + B(\tau)A_\theta(\theta; \tau)\sigma_{r,\theta} - B_\tau(\tau)r - A_\tau(\theta; \tau) - r = 0 \end{aligned}$$

and verify that for the previous equation to be uniformly satisfied over the domain of r we need

$$-B(\tau)(k + \lambda_1) + B^2(\tau) \frac{\sigma_1^2}{2} - B_\tau(\tau) - 1 = 0.$$

The solution for $B(\tau)$, subject to the initial condition that $B(0) = 0$, is well known (see Cox et al. (1985)),

$$B(\tau) = \frac{2(e^{\delta\tau} - 1)}{(\lambda_1 + \delta + k)(e^{\delta\tau} - 1) + 2\delta}, \quad (6)$$

$$\delta = \sqrt{(\lambda_1 + k)^2 + 2\delta_1^2}$$

which reduces to $B(\tau) = (1 - e^{-k\tau})/k$ when $\lambda_1 = \sigma_1^2 = 0$ (see Vasicek (1977)). If $m(\theta)$, $s(\theta)^2$, and $l(\theta)$ were linear functions of θ as postulated in equation (3'), yields would be linear in r and θ , and bond prices would have the familiar form

$$P(r, \theta; \tau) = e^{-C(\tau) - B(\tau)r - D(\tau)\theta} \quad (7)$$

where $A(\theta; \tau) = C(\tau) + D(\tau)\theta$. Conditions for yields to be linear in the underlying state variables are given in Cox et al. (1981) and Duffie and Kan (1993).

In the following we show how, by choosing appropriate weights, one can build a linear combination of two bond yields which is independent of r and thus mimics the variation of the second factor only. This implication that yields of different maturities can be used as instruments for unobservable factors is well known (Cox et al. (1985, pp. 398–401)), and is the basis of a number of empirical studies, such as Stambaugh (1988) and Sun (1992), before ours.

Formally, consider two bonds of maturity τ_1 and τ_2 , respectively. From equation (5), the corresponding yields are

$$y(r, \theta; \tau_i) = - \frac{\ln P(r, \theta; \tau_i)}{\tau_i} = \frac{A(\theta; \tau_i) + B(\tau_i)r}{\tau_i}, \quad (8)$$

$$i = 1, 2.$$

Solving for r from the first yield, substituting into the second, and rearranging we obtain

$$\begin{aligned} \tau_1 B(\tau_2) y(r, \theta; \tau_1) - \tau_2 B(\tau_1) y(r, \theta; \tau_2) \\ = B(\tau_2) A(\theta; \tau_1) - B(\tau_1) A(\theta; \tau_2). \end{aligned} \quad (9)$$

Note that this quantity does not depend on r . Thus we can use an affine function of the variable on the left-hand side of equation (9) as a proxy for θ . In fact, if the drift and diffusion of the process for θ were also linear in θ , then prices would be of the form in equation (7), and the second factor θ could be written as

$$\theta = \frac{(B(\tau_2)[\tau_1 y(r, \theta; \tau_1) - C(\tau_1)] - B(\tau_1)[\tau_2 y(r, \theta; \tau_2) - C(\tau_2)])}{B(\tau_2)D(\tau_1) - B(\tau_1)D(\tau_2)}. \quad (10)$$

While this is the case only when θ is an exact affine function of the variable on the left-hand side of equation (9), equation (10) justifies a proxy for θ , which is used in the empirical analysis of the next section. We denote this proxy by $\hat{\theta}$

$$\hat{\theta} = a_0 + a_1 [B(\tau_2)\tau_1 y(r, \theta; \tau_1) - B(\tau_1)\tau_2 y(r, \theta; \tau_2)]. \quad (11)$$

Using the proxy (11) rather than the variable (10) is convenient in that even if m and s are not linear in θ , we would still expect equation (11) to be a reasonable approximation of the true, unknown, functional form relating θ to any two bond yields. At this point we may also note that if the conditional mean and variance of the process for θ were in fact linear in θ , all proxies estimated from any bond-maturity pair (τ_1, τ_2) would be the same.

IV. Empirical Analysis

This section uses the previous theoretical findings to show how an equilibrium term-structure model can be used to learn about the dynamics of the short-term rate.

An important issue in estimation is that of identifying empirical counterparts to the factors r and θ , which are in principle unobservable. We follow a standard practice (see, for example, Chan et al. (1992) and Longstaff and Schwartz (1992)) and treat the one-month Treasury bill rate as a proxy for the instantaneous rate r . While this practice is convenient, the one-month rate is only an approximation of the instantaneous rate of interest, because yields on any finite-maturity bond depend on *both* factors, r and θ , as well as on

their risk premiums $\lambda_0 + \lambda_1 r$ and $l(\theta)$ (see, for example, Pearson and Sun (1994)). Second, we proxy θ with the quantity proposed in equation (11). The approximation depends on the fact that *only* when m and s are linear, is θ a linear function of the yields in equation (11).

A second issue is that we need to discretize the stochastic differential equation describing dr to implement it on discretely sampled data. When there is only one factor as, for example, when θ is constant, we could estimate an *exact* stochastic difference equation for the instantaneous rate implied by the differential equation for dr . However, since we allow θ to vary, we shall use the Euler discretization often used in the literature (see, for example, Chan et al. (1992)),

$$r_{t+h} - r_t \approx kh(\theta_t - r_t) + \sqrt{\sigma_0^2 + \sigma_1^2 r} \epsilon_{t+h}. \tag{12}$$

Since we use monthly observations, we set $h = 1/12$ and estimate our model by maximizing the log-likelihood function

$$-0.5 \sum \left[\ln \sigma_{t+1}^2 + \frac{(r_{t+1} - \bar{r}_{t+1})^2}{\sigma_{t+1}^2} \right] \tag{13}$$

where

$$\bar{r}_{t+1} = \left(1 - \frac{k}{12}\right)r_t + \left(\frac{k}{12}\right)\hat{\theta}_t \tag{14}$$

$$\sigma_{t+1}^2 = \frac{\sigma_0^2}{12} + \left(\frac{\sigma_1^2}{12}\right)r_t \tag{15}$$

and $\hat{\theta}_t$ is given by

$$\hat{\theta}_t = a_0 + a_1 [B(\tau_2)\tau_{1y}(r_t, \theta_t; \tau_1) - B(\tau_1)\tau_{2y}(r_t, \theta_t; \tau_2)] \tag{16}$$

where

$$B(\tau) = \frac{2(e^{\delta\tau} - 1)}{(\lambda_1 + \delta + k)(e^{\delta\tau} - 1) + 2\delta}, \tag{17}$$

$$\delta = \sqrt{(\lambda_1 + k)^2 + 2\sigma_1^2}.$$

We estimate the model (6), (11), (13), (14), and (15) using monthly observations. The estimates presented here are obtained using one- and two-year bond yields to construct the proxy in equation (11): $\tau_1 = 1$ and $\tau_2 = 2$.³ The estimation period goes from July 1952 to December 1993. Results are presented in table 3.

³ We experimented with different maturity pairs as well, with very similar, if slightly worse, results. The robustness of our results to the choice of maturity pairs is further discussed in section V.

TABLE 3.—MAXIMUM-LIKELIHOOD ESTIMATION

Model	k	a_0	a_1	σ_0	σ_1	Log-Likelihood
<i>July 1952–December 1993</i>						
Vasicek	0.383 (0.107)	0.051 (0.013)		0.025 (0.000)		2202
CIR	0.296 (0.096)	0.051 (0.011)			0.102 (0.001)	2291
Vasicek*	1.778 (0.228)	0.000 (0.007)	-2.068 (0.216)	0.024 (0.000)		2210
CIR*	2.122 (0.246)	-0.001 (0.002)	-2.269 (0.159)		0.099 (0.002)	2305
<i>July 1952–August 1971</i>						
Vasicek	0.354 (0.180)	0.035 (0.008)		0.013 (0.000)		1169
CIR	0.489 (0.211)	0.034 (0.008)			0.082 (0.002)	1158
Vasicek*	2.064 (0.521)	-0.004 (0.004)	-2.479 (0.359)	0.012 (0.000)		1173
CIR*	2.616 (0.524)	-0.003 (0.002)	-2.769 (0.387)		0.079 (0.002)	1166
<i>September 1971–September 1979</i>						
Vasicek	0.370 (0.497)	0.079 (0.024)		0.019 (0.000)		452
CIR	0.238 (0.412)	0.089 (0.057)			0.075 (0.003)	461
Vasicek*	0.389 (0.930)	0.044 (0.084)	-1.642 (7.100)	0.019 (0.000)		452
CIR*	0.243 (0.510)	0.095 (0.072)	0.369 (5.234)		0.075 (0.004)	461
<i>October 1979–September 1982</i>						
Vasicek	2.790 (1.535)	0.112 (0.013)		0.061 (0.007)		127
CIR	2.463 (1.475)	0.112 (0.014)			0.180 (0.024)	128
Vasicek*	2.381 (1.669)	0.160 (0.159)	1.130 (3.497)	0.060 (0.007)		127
CIR*	1.924 (1.496)	0.189 (0.181)	1.670 (3.575)		0.180 (0.024)	128
<i>October 1982–December 1993</i>						
Vasicek	0.737 (0.410)	0.055 (0.010)		0.026 (0.001)		593
CIR	0.772 (0.458)	0.055 (0.011)			0.113 (0.004)	587
Vasicek*	3.986 (0.620)	-0.011 (0.008)	-3.580 (0.597)	0.023 (0.001)		604
CIR*	4.467 (0.518)	-0.013 (0.008)	-4.108 (0.612)		0.102 (0.004)	602

Note: Standard errors are in parentheses. $\lambda_1 = 0$.

The table shows parameter estimates for four different models:

1. Vasicek: $a_1 = \sigma_1 = 0$
2. CIR: $a_1 = \sigma_0 = 0$

3. Vasicek*: $\sigma_1 = 0$
4. CIR*: $\sigma_0 = 0$

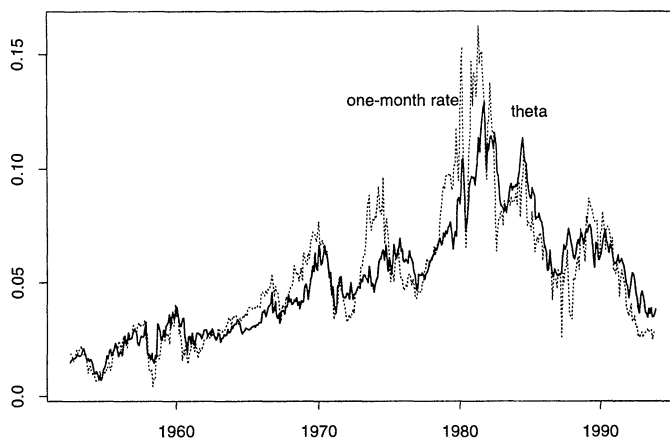
In Vasicek and CIR we restrict $\hat{\theta}$ to be a constant. The first model corresponds to the Vasicek (1977) Ornstein–Uhlenbeck process, while the second model corresponds to the Cox et al. (1985) square-root process. In Vasicek and CIR the mean of the square-root process is also affected by $\hat{\theta}$.

By equations (11) and (6) the coefficients relating the central-tendency proxy to bond yields depend on the parameter λ_1 which, in turn, describes the sensitivity of the market price of r -risk to r . In the empirical implementation, though, we found the log-likelihood to be largely insensitive to different values of λ_1 : the Vasicek* and CIR* models were estimated imposing $\lambda_1 = 0.0, -0.5, -1.0, -2.0$, with slight decreases in likelihood value the larger the λ_1 is in absolute value. Parameter estimates and the extracted $\hat{\theta}$ series were also essentially the same. Table 3 reports the estimation results for the different models assuming $\lambda_1 = 0$.

The full-period estimation results indicate that in the Vasicek* and CIR* models the estimate of a_1 is negative and significant: the central-tendency proxy is indeed time varying. It is also apparent that the level of the one-month rate affects its conditional volatility: in the CIR* model the estimate of σ_1 is positive and significant. Also, it is worth noting that the assumption of a constant central tendency (Vasicek and CIR models) leads to lower estimates of the mean-reversion parameter k . This is consistent with the intuition that interest rates should converge more quickly toward a central tendency which is time varying than toward a constant one.

Turning now to the subperiod evidence, we find that during the two periods September 1971–September 1979 and October 1979–September 1982, the measure of the central tendency captures little if any time variation in the interest rate. This evidence is in agreement with the findings presented in table 2, which shows that for the second and third subperiods the level and the changes of longer maturity yields have little explanatory power for the future level and changes of the one-month rate, respectively. A possible interpretation for these findings is that during the September 1971–September 1979 period the one-month rate itself was changing very smoothly, making the very concept of a central tendency of little use, at least at the monthly frequency. Also, the Federal Reserve’s aggressive management of nonborrowed reserves during the October 1979–September 1982 period resulted in such a high volatility of interest rates that the notion of a smoothly changing time-varying central tendency is again useless. For this period, only the conditional volatility parameters can be estimated with reasonable precision, and the data show a slight preference for a model where volatility is tied to the level of the interest rate (CIR and CIR*) relative to a model

FIGURE 1.—PLOT OF $\hat{\theta}$ (SOLID LINE) AND ONE-MONTH RATE (DASHED LINE)



The proxy for the central tendency $\hat{\theta}$ is calculated as in equation (11) using the one- and two-year bond yields and the estimates of the model parameters reported in table 3.

with constant volatility (Vasicek and Vasicek*). On the other hand, for the first and last subperiods the central-tendency proxy plays an important role in explaining the conditional mean of the one-month rate. These two periods have in common a somewhat “eclectic” approach of the Federal Reserve to monetary policy, where both interest rate volatility and the growth of monetary aggregates were taken into account when formulating operating targets.

Using the parameter estimates for the CIR* model of table 2, for the entire period, we obtain a time series of $\hat{\theta}$. Figure 1 plots $\hat{\theta}$ and the one-month rate. The visual evidence of figure 1 confirms that $\hat{\theta}$ indeed behaves as a central tendency, and it is less volatile than the one-month rate of interest.

Table 4, panel A, presents some summary statistics for the series $\hat{\theta}$ estimated for the entire period and for the four subperiods, in both levels and first differences. A comparison between table 4 and table 1 shows that $\hat{\theta}$ has essentially the same mean as the one-month rate, while its variability is always lower (both for the levels and for the first differences). This is not surprising since the central tendency should fluctuate around the same long-run mean as the one-month rate, and should exhibit a smoother behavior than the rate that reverts toward it. Other indications that $\hat{\theta}$ displays a smoother behavior than the one-month rate are as follows. The autocorrelation coefficient of $\hat{\theta}$ is always higher than that of the one-month rate. Moreover, when we consider the entire period, the kurtosis of the first differences of $\hat{\theta}$ is much lower than that of the first differences of the one-month rate: we may conclude that jumps in the central tendency are unlikely and our estimation of $\hat{\theta}$ acts as a “filter” of the one-month rate, eliminating some of the outliers in that series.

Table 4, panel B, calculates the correlations between $\hat{\theta}$ series, extracted using different maturity pairs, using the CIR* model. We may recall that if the process for θ were in fact linear (that is, with $m(\theta)$ and $s(\theta)$ linear in equation (3)),

TABLE 4.—SUMMARY STATISTICS AND CORRELATIONS FOR ESTIMATED CENTRAL TENDENCY θ
A. Summary Statistics

Sample Period	Sample Mean	Standard Deviation	Skewness	Kurtosis	First-Order Autocorrelation
<i>Levels</i>					
June 1952–November 1993	5.092	2.522	0.634	-0.076	0.985
June 1952–July 1971	3.178	1.359	0.700	0.362	0.957
August 1971–August 1979	8.931	0.210	0.534	-0.128	0.832
September 1979–August 1982	11.114	1.113	0.211	-0.522	0.780
September 1982–November 1993	5.988	1.954	0.231	-0.394	0.961
<i>First Differences</i>					
July 1952–November 1993	0.000	0.003	-0.150	2.963	0.013
July 1952–July 1971	0.000	0.003	0.180	4.029	-0.178
September 1971–August 1979	0.000	0.001	-1.119	7.045	-0.298
October 1979–August 1982	-0.000	0.006	0.543	0.049	0.121
October 1982–November 1993	-0.000	0.003	0.135	0.255	0.215
B. Correlation Between θ Series					
	$\hat{\theta}_{1,2}$	$\hat{\theta}_{1,3}$	$\hat{\theta}_{1,4}$	$\hat{\theta}_{1,5}$	
<i>Levels, June 1952–November 1993</i>					
	$\hat{\theta}_{1,2}$	1.000			
	$\hat{\theta}_{1,3}$	0.995	1.000		
	$\hat{\theta}_{1,4}$	0.992	0.997	1.000	
	$\hat{\theta}_{1,5}$	0.990	0.996	0.998	1.000
<i>First differences, July 1952–November 1993</i>					
	$\hat{\theta}_{1,2}$	1.000			
	$\hat{\theta}_{1,3}$	0.834	1.000		
	$\hat{\theta}_{1,4}$	0.804	0.888	1.000	
	$\hat{\theta}_{1,5}$	0.792	0.872	0.903	1.000

all proxies estimated from any pair of bond yields would be the same. The correlations for the levels are all very close to 1, showing that different maturity pairs lead to very similar θ series. Still, the correlations for the first differences of the θ series are lower, which is consistent, for example, with departures from linearity in the process for θ .

V. Diagnostics of Model

In this section we perform some tests of the appropriate specification of our model. First, we test whether the spread $\theta - r$ indeed helps predict changes in r , as implied by our model. Second, we test whether the one-month rate helps predict future changes in the central-tendency proxy, which should not be the case according to the assumed recursive structure of the model. Third, we compare the forecast of changes in the one-month rate based on our model to an unrestricted forecast.

In table 5, panel A, we estimate the model

$$r_{t+1} - r_t = \beta_0 + \beta_1 \hat{\theta}_t + \beta_2 r_t + \text{error}_{t+1} \quad (16)$$

where $\hat{\theta}$ is obtained from the estimation of the CIR* model (see table 3). This model differs from the CIR* model in that we do not impose the restriction that $\beta_1 = -\beta_2 = k/12$. Moreover, we are not imposing a specific structure for the conditional volatility of the innovation. The standard errors

reported in the table are adjusted for heteroskedasticity of unknown form of the residuals. The evidence of panel A is supportive of our analysis. The parameters are precisely estimated for the entire sample and in the last subperiod. Also, with the exclusion of the first subperiod, we cannot reject the restriction that $\beta_1 = -\beta_2$. This is consistent with the prediction of our model that the spread between the estimated central-tendency proxy and the one-month rate, $\hat{\theta} - r$, has explanatory power for future changes in the one-month rate.

The second test is reported in table 5, panel B, where we estimate the model

$$\hat{\theta}_{t+1} - \hat{\theta}_t = \beta_0 + \beta_1 \hat{\theta}_t + \beta_2 r_t + \text{error}_{t+1}. \quad (17)$$

According to the assumed recursive structure of our model (see equations (2) and (3)), r should not help predict future changes in $\hat{\theta}$. Panel B provides evidence that this assumption is not always consistent with the data. While in the last two subperiods β_2 is close to zero and imprecisely estimated, which is consistent with our model, the estimate of β_2 is positive and precise for the entire period and for the first subperiod. The fit of the model (17), as measured by the adjusted R^2 , is best for the third period, when, as shown in table 4, θ displays the lowest serial correlation.

Table 6 presents one last diagnostic of our model: a comparison between the optimal forecast of $r_{t+1} - r_t$ implied

TABLE 5.—MODELS SPECIFICATIONS
A. DOES $\hat{\theta} - r$ EXPLAIN dr ?

Sample Period	β_1	β_2	Adjusted R^2	Durbin-Watson Statistic	$\chi^2(1)$
July 1952–December 1993	0.128 (0.049)	-0.133 (0.049)	0.044	2.012	0.053 (0.816)
July 1952–August 1971	0.152 (0.056)	0.000 (0.021)	0.047	2.256	4.505 (0.033)
September 1971–September 1979	-0.120 (0.454)	-0.022 (0.030)	-0.010	2.206	0.099 (0.752)
October 1979–September 1982	0.121 (0.319)	-0.201 (0.155)	0.047	1.733	0.032 (0.856)
October 1982–December 1993	0.314 (0.079)	-0.325 (0.079)	0.162	2.151	0.172 (0.677)

B. Do $\hat{\theta}$ and r Explain $d\hat{\theta}$?

Sample Period	β_1	β_2	Adjusted R^2	Durbin-Watson Statistic
July 1952–November 1993	-0.072 (0.021)	0.055 (0.020)	0.031	1.942
July 1952–July 1971	-0.158 (0.044)	0.119 (0.040)	0.044	2.204
September 1971–August 1979	-0.054 (0.033)	0.003 (0.008)	0.020	2.551
October 1979–August 1982	-0.259 (0.087)	-0.042 (0.046)	0.126	1.475
October 1982–November 1993	-0.036 (0.037)	0.017 (0.035)	-0.000	1.550

Notes: We estimate the following model:

$$r_{t+1} - r_t = \beta_0 + \beta_1 \hat{\theta}_t + \beta_2 r_t + \text{error}_{t+1}. \quad (16)$$

The proxy for the central tendency $\hat{\theta}$ is calculated as in equation (11) using the one- and two-year bond yields and the estimates of the model parameters reported in table 3. Standard errors, adjusted for heteroskedasticity (White (1980)), are in parentheses. $\chi^2(1)$ is the Wald test of the restriction that $\beta_1 = -\beta_2$; the statistic is distributed chi-square with one degree of freedom, with p -value indicated in parentheses below.

by the theoretically motivated CIR* model, and an unrestricted forecast. Based on our model, we have

$$\begin{aligned} E_t r_{t+1} - r_t &= -\left(\frac{k}{12}\right) r_t + \left(\frac{k}{12}\right) \hat{\theta}_t \\ &= a_0 \left(\frac{k}{12}\right) + \left(-\frac{k}{12}\right) r_t + a_1 \left(\frac{k}{12}\right) B(2) y_t(\tau_1) \\ &\quad + (-a_1) \left(\frac{k}{12}\right) B(1) 2 y_t(\tau_2) \\ &= w_0 + w_r r_t + w_{y(1)} y_t(1) + w_{y(2)} y_t(2). \end{aligned}$$

These weights are compared to the estimated coefficients of the model

$$\begin{aligned} r_{t+1} - r_t &= \beta_0 + \beta_1 r_t + \beta_2 y_t(1) \\ &\quad + \beta_3 y_t(2) + \text{error}_{t+1}. \end{aligned} \quad (18)$$

Panel A of table 6 presents the estimates of w_r , $w_{y(1)}$, and $w_{y(2)}$. For the periods in which the time-varying central tendency matters most, namely, the entire period and the first and last subperiods, the estimate of $w_{y(1)}$ is negative, while

the estimate of $w_{y(2)}$ is positive and larger in absolute size. This is consistent with the notion that a measure of the slope of the yield curve, $y(2) - y(1)$, should anticipate changes in the one-month rate in the same direction. Also, we calculate the sum of the three weights, and we find that it is quite close to zero for the entire period, and for the second and third subperiods. This is not surprising, since we are using a linear combination of three series, r , $y(1)$, and $y(2)$, which are highly persistent (see table 1), to explain the changes in the one-month rate, which instead exhibit almost no serial correlation (see table 1). Hence the coefficients of the linear combination need to sum up to zero. Also, if we believe that the three interest rate series contain a unit root, this would be evidence that they are *cointegrated*, where the weights w represent the coefficients of the cointegrating vector. Panel B of table 6 presents the estimates of the parameters of the unrestricted model (18). While the estimates of the coefficients β_1 , β_2 , and β_3 are quite different from those of the weights w , the general message from the unrestricted model is similar to that of the restricted one. First the estimates of β_3 are not significant, which implies that the one-month rate and one longer maturity yield are sufficient to model the conditional expectation of $r_{t+1} - r_t$. Hence this confirms that

TABLE 6.—RESTRICTED AND UNRESTRICTED FORECASTS OF SHORT-TERM RATE CHANGES
A. RESTRICTED FORECAST

Sample Period	w_r	$w_{y(1)}$	$w_{y(2)}$	Σw
July 1952–December 1993	-0.176 (0.020)	-0.186 (0.056)	0.332 (0.027)	-0.030 (0.033)
July 1952–August 1971	-0.218 (0.046)	-1.052 (0.146)	1.961 (0.078)	0.691 (0.077)
September 1971–September 1979	-0.020 (0.042)	0.583 (0.157)	-0.655 (0.186)	-0.092 (0.032)
October 1979–September 1982	-0.160 (0.124)	0.844 (0.155)	-1.476 (0.623)	-0.092 (0.468)
October 1982–December 1993	-0.372 (0.043)	-0.919 (0.188)	1.817 (0.154)	0.526 (0.063)

B. UNRESTRICTED FORECAST				
Sample Period	β_1	β_2	β_3	$\Sigma \beta_i$
July 1952–December 1993	-0.436 (0.079)	0.673 (0.160)	-0.283 (0.125)	-0.046 (0.018)
July 1952–August 1971	-0.337 (0.072)	0.361 (0.152)	-0.068 (0.146)	-0.044 (0.020)
September 1971–September 1979	-0.318 (0.168)	0.469 (0.486)	-0.161 (0.379)	-0.009 (0.045)
October 1979–September 1982	-0.371 (0.241)	0.748 (0.897)	-0.639 (0.771)	-0.261 (0.144)
October 1982–December 1993	-0.568 (0.108)	0.797 (0.256)	-0.288 (0.215)	-0.060 (0.026)

Notes: According to equations (14) and (11), the optimal forecast of short-term rate changes in a linear combination of the short-term rate and two longer maturity bond yields,

$$\begin{aligned}
 E_{t+1}r_t - r_t &= -(k/12)r_t + (k/12)\theta, \\
 &= a_0(k/12) + (-k/12)r_t + a_1(k/12)B(\tau_1)\tau_1 y_t(\tau_1) \\
 &\quad + (-a_1)(k/12)B(\tau_1)\tau_2 y_t(\tau_2) \\
 &= w_0 + w_r r_t + w_{y(1)} y_t(\tau_1) + w_{y(2)} y_t(\tau_2)
 \end{aligned}$$

where w_x denotes the estimate of the weight (loading) on the explanatory variable x . Each weight is a function of the estimated parameters and thus an estimate itself. Hence we compute standard errors on w_x using a first-order Taylor series expansion around the ML estimates, and the estimated variance-covariance matrix of the ML estimates of the parameters of the model. We contrast the estimates of the weights w with the estimates of the regression parameters of the unrestricted model,

$$r_{t+1} - r_t = \beta_0 + \beta_1 r_t + \beta_2 y_t(\tau_1) + \beta_3 y_t(\tau_2) + \text{error}_{t+1}.$$

Standard errors are in parentheses and are adjusted for heteroskedasticity (White (1980)).

two factors (interest rates) are sufficient to capture the dynamics of the short-term rate. Note, though, that the estimates of β_1 are larger (in absolute value) than the estimates of the weight w_r . This is further evidence that the short-term rate contributes to explain the time-varying central tendency θ , contrary to the assumed recursive structure of our model. Second, the estimates of β_1 are negative, while the estimates of β_2 are positive. This is again consistent with the notion that a measure of the slope of the yield curve, this time $y(1) - r_t$, should anticipate changes in the same direction in the one-month rate. Third, the sum of the three β coefficients is always close to zero, which is the same as was obtained for the restricted model.

VI. Conclusions

In one-factor models, such as Cox et al. (1985) or Vasicek (1977), the conditional mean of the instantaneous rate changes with its current level. This paper gathers evidence that the conditional mean of the one-month rate is explained by bond yields of different maturities, even after controlling for the effect of the current level of the one-month rate. This suggests the presence of a second factor driving the conditional mean of the one-month rate, other than the level of the one-month rate itself: we refer to this second factor as the *central tendency*. The above idea is captured in a two-factor model of the term structure, where the instantaneous rate fluctuates around a stochastic central tendency. We then build a proxy for the central-tendency factor based on the information contained in the term structure of interest rates. Based on this central-tendency proxy, we estimate a model of the one-month rate which performs better than models that assume the central tendency to be constant.

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