Correction of typo in "Portfolio Optimization with Mental Accounts" *Journal of Financial and Quantitative Analysis* Vol. 45, No. 2, Apr. 2010, pp. 311–334

by Sanjiv Das, Harry Markowitz, Jonathan Scheid, Meir Statman

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The captioned article contained a typo which was recently uncovered in two equations (18) and (19), as part of a feasibility check on the solution to the problem in this paper. The typo arises from an incorrect algebraic derivation in the last step of the feasibility check. No results in the paper are affected by this typo, as this was presented as an algebraic approach and all solution checking in the paper was undertaken numerically. There is no logical error in the feasiblity check and the error is purely typographical.

The error occurs in page 324 of the paper, which is reproduced in Figure 1. The correction is that equation (18) is presented as follows:

$$w = \frac{1}{\Phi^{-1}(\alpha)M}\,\Sigma^{-1}\left[\mu - \left(\frac{\mathbf{1}\Sigma^{-1}\mu + \Phi^{-1}(\alpha)M}{\mathbf{1}\Sigma^{-1}\mathbf{1}}\right)\mathbf{1}\right]$$

but should be corrected to:

$$w = \frac{1}{\Phi^{-1}(\alpha)M}\,\Sigma^{-1}\left[\left(\frac{\mathbf{1}\Sigma^{-1}\mu + \Phi^{-1}(\alpha)M}{\mathbf{1}\Sigma^{-1}\mathbf{1}}\right)\mathbf{1} - \mu\right]$$

And the same error carries forward to equation (19) which is presented in the paper as:

$$w = \frac{1}{\Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}\,\Sigma^{-1}\left[\mu - \left(\frac{\mathbf{1}\Sigma^{-1}\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}{\mathbf{1}\Sigma^{-1}\mathbf{1}}\right)\mathbf{1}\right]$$

but should be corrected to:

$$w = \frac{1}{\Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}\,\Sigma^{-1}\left[\left(\frac{\mathbf{1}\Sigma^{-1}\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}{\mathbf{1}\Sigma^{-1}\mathbf{1}}\right)\mathbf{1} - \mu\right]$$

The two terms in the large square brackets are to be interchanged. The rest of the equations (9)-(17) remain unaffected.

B. Feasibility in MA

Achieving particular combinations of thresholds, probabilities of failing to reach them, and expected returns may not always be feasible with a given set of assets. The MA problem has a feasible solution when

$$H \leq w'\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{1/2}. \tag{9}$$

The problem has no *feasible* solution when $H > w'\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{1/2}$. One way to find if the problem has a feasible solution is to maximize the value of the right-hand side of equation (9) and check if it is greater than H. This results in the following optimization program:

$$\max_{w} Q \equiv w'\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{1/2}, \tag{10}$$

subject to

$$w'\mathbf{1} = 1. \tag{11}$$

The Lagrangian for this problem is

$$\max_{w,\lambda} Q = w'\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{1/2} + \lambda[1 - w'\mathbf{1}]. \tag{12}$$

The first-order conditions are

$$\frac{\partial Q}{\partial w} = \mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}\Sigma w - \lambda\mathbf{1} = 0, \tag{13}$$

$$\frac{\partial Q}{\partial \lambda} = 1 - w'\mathbf{1} = 0. \tag{14}$$

We premultiply all terms in equation (13) by Σ^{-1} and, defining $[w'\Sigma w]^{-1/2} \equiv M$, we get, after rearranging,

$$\lambda\Sigma^{-1}\mathbf{1} = \Sigma^{-1}\mu + \Phi^{-1}(\alpha)Mw, \tag{15}$$

$$\lambda\mathbf{1}'\Sigma^{-1}\mathbf{1} = \mathbf{1}'\Sigma^{-1}\mu + \Phi^{-1}(\alpha)M\mathbf{1}'w. \tag{16}$$

Noting that $\mathbf{1}'w = 1$, we get

$$\lambda = \frac{\mathbf{1}'\Sigma^{-1}\mu + \Phi^{-1}(\alpha)M}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}. \tag{17}$$

Substitute the solution for λ into equation (15) and rearrange to get the equation for portfolio weights:

$$w = \frac{1}{\Phi^{-1}(\alpha)M}\Sigma^{-1}\left[\mu - \left(\frac{\mathbf{1}'\Sigma^{-1}\mu + \Phi^{-1}(\alpha)M}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)\mathbf{1}\right]. \tag{18}$$

Note however, we have eliminated λ but we have obtained an equation with $w \in R^n$ on both sides, since $M = [w'\Sigma w]^{-1/2}$, giving us

$$w = \frac{1}{\Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}\Sigma^{-1}\left[\mu - \left(\frac{\mathbf{1}'\Sigma^{-1}\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{-1/2}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)\mathbf{1}\right]. \tag{19}$$

This is a system of n implicit equations, best solved numerically. Once we get the solution and plug it back into the objective function $Q \equiv w'\mu + \Phi^{-1}(\alpha)[w'\Sigma w]^{1/2}$ to get the maximized value, we can check if $H < Q$. If not, then the problem is infeasible with the current portfolio choice set, and other assets need to be considered or H reduced.

Figure 1: Feasibility check on page 324. The typo is an error in presenting equations 18 and 19. The two terms in the large square bracket are to be interchanged.