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The surprise element: jumps in interest rates

Sanjiv R. Das *

Department of Finance, Leavey School of Business, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053-0388, USA

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Abstract

That information surprises result in discontinuous interest rates is no surprise to participants in the bond markets. We develop a class of Poisson–Gaussian models of the Fed Funds rate to capture surprise effects, and show that these models offer a good statistical description of short rate behavior, and are useful in understanding many empirical phenomena. Jump (Poisson) processes capture empirical features of the data which would not be captured by Gaussian models, and there is strong evidence that existing Gaussian models would be well-enhanced by jump and ARCH-type processes. The analytical and empirical methods in the paper support many applications, such as testing for Fed intervention effects, which are shown to be an important source of surprise jumps in interest rates. The jump model is shown to mitigate the non-linearity of interest rate drifts, so prevalent in pure-diffusion models. Day-of-week effects are modelled explicitly, and the jump model provides evidence of bond market overreaction, rejecting the martingale hypothesis for interest rates. Jump models mixed with Markov switching processes predicate that conditioning on regime is important in determining short rate behavior. © 2002 Elsevier Science S.A. All rights reserved.

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1. Introduction

> "Natura non facit saltum"– Nature does not Jump Alfred Marshall, title page, Principle of Economics, 1890.

* Tel.: +1-408-4385; fax: +1-408-554-4029.
E-mail address: srdas@scu.edu (S.R. Das).

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Surprise is an intrinsic element of financial markets and a simple fact of life. The fixed-income markets are no exception to this.

Despite vast amounts of available public information on the economy, and exhaustive quantities of research, surprises still occur with sufficient magnitude and regularity to substantially impact yields, bond prices and bid-ask spreads. Demand shocks such as market behavior at Treasury auctions often result in jumps, as do economic news announcements. Balduzzi et al. (1998c) examined 17 different types of economic announcements, and found that eight of them significantly impacted the market. Green (1998) also found that the release of information increases information asymmetry, impacting the Treasury market. Similar results were provided by Dwyer and Hafer (1989), and Hardouvelis (1988). Exogenous intervention in the markets by the Federal Reserve causes jumps. Supply shocks are another factor, as regular debt refundings inject sufficient volume to magnify price effects. As Merton (1976) emphasizes, routine trading information releases are well depicted by smooth changes in interest rates, yet bursts of information are often reflected in price behavior as jumps.

This research examines the role of jump-enhanced stochastic processes in modeling the Fed Funds rate. The paper offers three distinct sets of contributions. (1) We develop an analytical modeling framework for jumps in fixed-income markets.[1] (2) We establish that modeling surprises with jump based models provides a better statistical characterization of the short interest rate than is possible with complex Gaussian models. (3) We present a range of applications of the model to demonstrate that these models offer a rich habitat in which to characterize various bond market phenomena.

From a statistical point of view, there are three features of the short rate process that consistently exist in bond markets.

(1) *Higher moment behavior*: changes in interest rates demonstrate considerable skewness and kurtosis, which can substantially affect the pricing of derivative securities. Table 1 provides summary statistics for the short rate of interest. The presence of leptokurtosis in interest rate changes is undeniable and makes an initial case for jump models.[2]

[1] See Ahn and Thompson (1988), Attari (1997), Babbs and Webber (1995), Backus et al. (1997), Burnetas and Ritchken (1996), Chacko (1998), Chacko and Das (1998), Das and Foresi (1996), Duffie and Kan (1996), Duffie et al. (2000), Heston (1995), Naik and Lee (1993), Piazzesi (1998), Shirakawa (1991), Van Steenkiste and Foresi (1999) for a range of jump-diffusion theoretical models.

[2] See Backus et al. (1997) for an excellent exposition of why jumps may better explain the high degree of curvature in yield curves.

Table 1
Descriptive statistics[a]

Statistic	r	dr
Mean	5.8100	−0.0005
Standard deviation	1.9558	0.2899
Skewness	0.3032	0.3950
Excess kurtosis	−0.8304	19.8667
Minimum	2.58	−2.70
Maximum	10.71	2.83

[a]The table presents descriptive statistics for the Fed Funds rate over the period January 1988 to December 1997. The data is daily in frequency. The statistics reported are for the interest rate level (r) and the change in interest rates (dr).

(2) *Volatility behavior*: short rate volatility is very high, and is persistent.[3] By enhancing jump models with ARCH features and regime switches, this aspect is captured.
(3) *Autocorrelation and mean reversion*: interest rates evidence both, which complicates the assessment of information effects. Mean reversion may arise naturally from underlying macroeconomic currents, or from corrections on account of bond market overreaction. By modeling information surprises via a jump, these effects are separated in the paper.

Using a set of analyses on Fed Funds data, we conclude that mixed models with time-varying volatility and jumps are predicated, because neither model stand-alone is able to fit the empirical distribution of the data appropriately. Previous work[4] employing time-varying volatility models, such as ARCH processes, offers strong evidence that time varying volatility models provide a good empirical fit. We find that significant improvements are obtained when jumps are introduced into this class of models.

The model has many possible applications. After establishing that jumps play an essential role in describing interest rate dynamics, we explore *seven* different empirical phenomena using various jump models. First, information effects are captured easily in our models. As in Balduzzi et al. (1998c), we too find that volatility is significantly higher after a news release.

Second, we examine the pattern of higher-order moments of the short rate process, analytically and empirically. Conditional skewness and kurtosis varies with the time interval between data observations, and results in distinctly different patterns for time-varying volatility models versus jump models. Poisson–Gaussian models better match conditional and unconditional skewness and kurtosis at varying maturities than time-varying models without jumps.

[3] Coleman et al. (1993) find that in the 1980s, the standard deviation of monthly changes in the short (1-month) rate was 128 basis points. Duffee (1996) discusses the idiosyncratic variation in short rates.

[4] See Brenner et al. (1996) and Koedijk et al. (1996).

Third, Federal Reserve activity impacts the short rate via information surprises. Using Fed Open Market Committee meeting dates in our jump model shows that two-day meetings of the FOMC have a substantially greater surprise component than one-day meetings. [5]

Fourth, we are able to analyze day-of-week effects using the model. Jumps appear to occur more frequently on Wednesdays, probably on account of option expiry effects.

Fifth, there is a growing literature on the non-linearity of the drift in interest rates, which has cast doubts on the simple linear mean-reverting form used in most diffusion models (see Ait-Sahalia, 1996a, b; Chapman and Pearson, 2000; Stanton, 1997). Discussion of these issues was also taken up by Bodoukh et al. (1998), Ang and Bekaert (1998), and Duffee (1999). The addition of a jump process to the diffusion model ameliorates the non-linearity. The more appealing inference is that non-linearity is caused by information effects, which, when captured via a jump model, resurrect simpler linear drift models.

Sixth, we examine overreaction in the bond markets (as in Hamilton, 1996). A specialized version of the model in the paper allows us to detect continuation and reversal effects, by examining their impact on jump probabilities. We find evidence in favor of both effects, implying that Fed funds rates are not martingales. The reversal effect is predominant, evidencing overreaction.

Finally, in a managed interest rate regime, the jump behavior in short rates comprises of two components: (a) infrequent changes in target rates by the Fed, and (b) frequent deviations from the targets, which are market driven (see Balduzzi et al., 1997, 1998a). An extension of the Poisson–Gaussian model to encompass regime-switches offers additional evidence in support of this empirical phenomena. (More supporting evidence was also presented by Naik and Lee (1993), Gray (1996), and Piazzesi (1998)). We find that the short rate process sometimes inhabits a high interest rate regime, where discontinuous behavior predominates, yet is more frequently in a low rate regime, evidenced by smooth rate transitions. We conclude that conditioning on the regime is important in determining the choice of jump model.

The detailed content of the paper comprises three sections, theoretical, empirical and applications. Section 2 provides a menu of jump-diffusion analytics. Section 3 contains an empirical assessment of jump models. Jumps are shown to account for a large part of the total variation in interest rates, and capture the patterns of higher-order moments which cannot be generated by diffusion models with time varying volatility alone. We demonstrate the strong complimentarity of jump and diffusion stochastic process choices in modeling the term structure. In Section 4, we present a series of applications,

[5] For recent work in this domain, see Piazzesi (1999).

demonstrating the usefulness of jump models in relation to the empirical phenomena described above. Finally, Section 5 offers conclusions and ideas for future research.

2. Model specification

The analytics for stochastic processes governing interest rates are more complex than that usually encountered for equities and foreign exchange rates. Mean reversion is one source of analytical complexity in models with jumps. It makes the probability function for the stochastic process dependent on the timing of jumps as well as their size (without mean reversion, the density function might depend on jump size only). There are also very few known solutions to the stochastic differential equations governing mean reverting jump-diffusions. These complexities make econometric analyses harder to implement. In this section, we develop the primary analytics for our econometric specifications; in later sections, we present extensions.

2.1. The basic stochastic process

The following is the mean reverting process for interest rates employed in this paper:

$$\mathrm{d}r = k(\theta - r)\,\mathrm{d}t + v\,\mathrm{d}z + J\,\mathrm{d}\pi(h), \tag{2.1}$$

where θ is a central tendency parameter for the interest rate r, which reverts at rate k. Therefore, the interest rate evolves with mean-reverting drift and two random terms, one a diffusion and the other a Poisson process embodying a random jump J. The variance coefficient of the diffusion is v^2 and the arrival of jumps is governed by a Poisson process π with arrival frequency parameter h, which denotes the number of jumps per year. The jump size J may be a constant or drawn from a probability distribution. The diffusion and Poisson processes are independent of each other, and independent of J as well. The discretized version of this process is used for estimation, and the details are presented in the following section. We assume that the coefficients are bounded, and that sufficient restrictions exist on the drift, diffusion and jump coefficients that give a unique, strong solution to Eq. (2.1).

2.2. The characteristic function

Assume that we are at time $t = 0$, and that we are looking ahead to time $t = T$. We are interested in the distribution of $r(T)$ given the current value of the interest rate $r(0) \equiv r_0 = r$. In order to derive the T-interval characteristic function $F(r, T; s)$ for the process (2.1), (s is the characteristic function parameter) we solve its Kolmogorov backward equation (KBE) subject to the boundary condition that

$$F(r, T = 0; s) = \exp(\mathrm{i}sr), \tag{2.2}$$

where $\mathrm{i} = \sqrt{-1}$. The backward equation is

$$0 = \frac{\partial F}{\partial r} k(\theta - r) + \frac{1}{2} \frac{\partial^2 F}{\partial r^2} v^2 - \frac{\partial F}{\partial T} + h\mathrm{E}[F(r + J) - F(r)]. \tag{2.3}$$

The last term comes from the effect of the Poisson shock. The solution (comprehensive details of the derivation for a much more general case are available in Duffie et al. (2000), and are thus not provided here) is provided below:

$$\begin{aligned} F(r, T; s) &= \exp[A(T; s) + rB(T; s)], \\ A(T; s) &= \int (k\theta B(T; s) + \tfrac{1}{2} v^2 B(T; s)^2 + h\mathrm{E}[\mathrm{e}^{JB(T;s)} - 1])\,\mathrm{d}T, \\ B(T; s) &= \mathrm{i}s \exp(-kT). \end{aligned} \tag{2.4}$$

Given the characteristic function, we can obtain the moments and the probability density functions for any choice of jump distribution where the jump intensity or distribution do not depend on the state variables.

The class of models in Eq. (2.4) are well-known and are termed exponential-affine models. For the original definition of the term, see Duffie and Kan (1996). Also for affine jump-diffusion models, see Das and Foresi (1996), Chacko (1998), Chacko and Das (1998), Duffie et al. (2000), Pan (1999). Exponential-affine models are those where the yield to maturity on all bonds may be written as a linear function of the state variables. These models have been thoroughly analyzed by Duffie et al. (2000) where state-dependence of the intensity has been provided for. The paper provides an empirical complement to this literature. At the outset, the paper assesses the basic Gaussian and Poisson–Gaussian models. We then show that by mixing a jump model with time-varying volatility processes, the fit of the model improves dramatically. Relaxation of the requirement of constant jump intensity also betters the fit. Finally, we examine a regime-switching model over two jump regimes.

2.3. The moments

A full derivation of the moments is provided in the appendix for completeness. To obtain the moments, we differentiate the characteristic function successively with respect to s and then find the value of the derivative when $s=0$. Let μ_n denote the nth moment, and F_n be the nth derivative of F with respect to s, i.e. $F_n = \partial^n F/\partial s^n$. Then $\mu_n = 1/i^n[F_n \mid s=0]$. Likewise $\mathrm{E}[J^n]$ denotes the nth moment of the jump shock. The first four moments (as derived in the appendix) are:

$$\mu_1 = \left(\theta + \frac{h\mathrm{E}[J]}{k}\right)(1-\mathrm{e}^{-kT}) + r\mathrm{e}^{-kT},$$

$$\mu_2 = \frac{v^2 + h\mathrm{E}[J^2]}{2k}(1-\mathrm{e}^{-2kT}) + \mu_1^2,$$

$$\mu_3 = h\mathrm{E}[J^3]\left(\frac{1-\mathrm{e}^{-3kT}}{3k}\right) + 3\mu_1(v^2 + h\mathrm{E}[J^2])\left(\frac{1-\mathrm{e}^{-2kT}}{2k}\right) + \mu_1^3,$$

$$\begin{aligned}\mu_4 = {}& h\mathrm{E}[J^4]\left(\frac{1-\mathrm{e}^{-4kT}}{4k}\right) + 3\left((v^2 + h\mathrm{E}[J^2])\left(\frac{1-\mathrm{e}^{-2kT}}{2k}\right)\right)^2 \\ &+ 4\mu_1 h\mathrm{E}[J^3]\left(\frac{1-\mathrm{e}^{-3kT}}{3k}\right) \\ &+ 6\mu_1^2\left((v^2 + h\mathrm{E}[J^2])\left(\frac{1-\mathrm{e}^{-2kT}}{2k}\right)\right) + \mu_1^4.\end{aligned}$$

Given any jump distribution with finite moments, we can compute the values of $\mathrm{E}[J^n]$, $n=1$–4. Given the derived moments above, provided the coefficients of the stochastic process are bounded, we obtain well-behaved processes that admit probability functions.

3. Estimation

There is a growing literature on estimation methods for term structure models. Ait-Sahalia (1996a, b), Chapman and Pearson (2000), and Pritsker (1998) examined single factor models and found that affine specifications may be incorrect. See also Chapman et al. (1999), Conley et al. (1997) and Stanton (1997) for further work in this direction. There is a vast literature specifically devoted to the estimation of affine models: Balduzzi et al. (1998b, 1996), Chacko (1998), Dai and Singelton (2000a,b), Duffee (1999), Piazzesi (1998, 1999), Singleton (2001). In particular, Dai and Singelton (2000a,b) provide general methods for affine models. The papers of Ait-Sahalia (1999, 2000) deal with the estimation of general diffusion models via maximum-likelihood, especially those in the non-affine class.

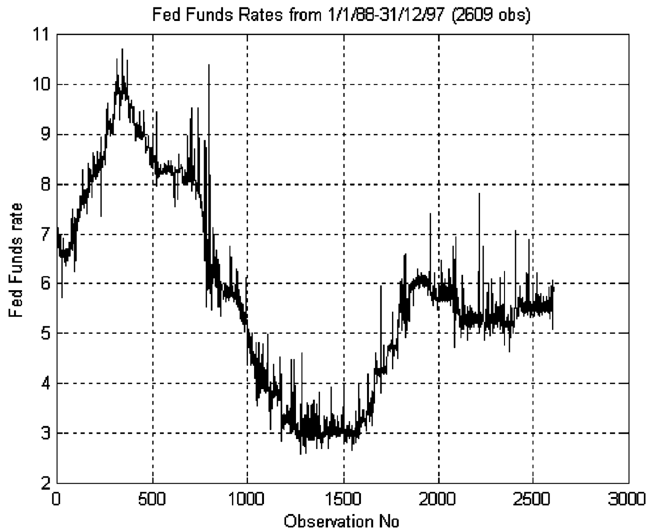


Fig. 1. A plot of the Fed Funds Rate for the period January 1988–December 1997. The data is daily and is obtained from the Federal Reserve. The total number of observations is 2609.

The estimation of jump-diffusion models has not been undertaken in the same earnest as pure-diffusion models. This paper explores the role of various mixed jump and time-varying volatility models, in the spirit of Naik and Lee (1993), especially developing a set of applications for this class of models. Probably, the earliest example of this genre, Naik and Lee (1993) developed a jump model for interest rates in which both the mean and volatility of the interest rate diffusion switched between regimes via a jump process. In addition to this paper, Babbs and Webber (1995) and Piazzesi (1999) also carry out the estimation of a jump model of interest rates. Chacko (1998) and Singleton (2001) develop characteristic function approaches to these models, both of which also deal with jump-diffusion models of interest rates. Ait-Sahalia (1998) provides technical conditions and tests for the presence of jumps in the continuous-time model when observing discrete data. [6]

Discrete-time analytics based on Section 2 are applied to daily data on the Fed funds rate for the period January 1988 to December 1997. The total number of observations is 2609. The data is from the Federal Reserve web site and is plotted in Fig. 1. In addition to the Fed Funds rate, other

[6] Outside the term structure literature, there is plenty of evidence for the existence of jumps in other economic variables. Ball and Torous (1983) and Jorion (1988) finds ample evidence for jumps in the equity and foreign exchange markets, and Bates (1996, 2000) does an extensive examination of equity and forex markets for combined jump and stochastic volatility models.

choices have often been made for the short rate proxy. Ait-Sahalia (1996a, b) used seven-day Eurodollar rates and Stanton (1997) used three-month Treasury bill rates. On the other hand, the paper by Chan et al. (1992) uses one-month T-bill yields, and Conley et al. (1997) use Fed funds rates. The data chosen for the paper takes into account the following considerations. First, by starting in 1988, we eliminate the period of the October 1987 crash, which would bias the results in favor of finding jumps. Second, the choice of short rate proxy is not always inconsequential. Chapman et al. (1999) show that for non-affine models, severe estimation biases may arise depending on the proxy chosen. However, their work shows that the biases are aggravated as the maturity of the rate chosen increases. Our use of the Fed funds rate should keep this bias lower than other choices such as the three month rate. The descriptive statistics for the data are in Table 1. An examination of the data reveals that changes in interest rates evidence a very high degree of kurtosis, a stylized fact that predicates the use of a jump model. Over the entire 10 year period, rates have quickly risen to a peak of 10% and then fallen to a low of 3%, finally stabilizing at a 6% level.

Our estimation exercise uses Poisson–Gaussian models extended for ARCH effects. They allow for mean-reversion in jump processes, and also test for the impact of Federal Reserve actions and day-of-the-week effects.

3.1. Estimator

In this section, a simple discrete-time approach allows us to estimate a model where the jumps are normally distributed. [7] We estimate the Poisson–Gaussian interest rate model using a Bernoulli approximation, first introduced in Ball and Torous (1983). The assumption made here is that in each time interval either only one jump occurs or no jump occurs. This is tenable for short frequency data, and may be debatable for data at longer frequencies. As Ball and Torous found, it provides an estimation procedure that is highly tractable, stable and convergent. Since the limit of the Bernoulli process is governed by a Poisson distribution, we can approximate the likelihood function for the Poisson–Gaussian model using a Bernoulli mixture of the normal

[7] It is now well recognized that discretization of continuous-time stochastic differential equations for estimation does introduce an estimation bias. However, this is small when the data is at daily frequency (Bergstrom, 1988). For a body of work on discrete time models and their continuous-time versions, see Backus et al. (1997, 1998a, b), and Sun (1992).

distributions governing the diffusion and jump shocks.[8] In discrete time, we express the process in Eq. (2.1) as follows:

$$\Delta r = k(\theta - r)\,\Delta t + v\,\Delta z + J(\mu,\gamma^2)\,\Delta\pi(q), \tag{3.2}$$

where v^2 is the annualized variance of the Gaussian shock, and Δz is a standard normal shock term. $J(\mu,\gamma^2)$ is the jump shock, which is normally distributed with mean μ and variance γ^2. $\Delta\pi(q)$ is the discrete-time Poisson increment, approximated by a Bernoulli distribution with parameter $q = h\,\Delta t + \mathrm{O}(\Delta t)$. We allow the variance v_t^2 to be ARCH in extending the Poisson–Gaussian model, and also permit the jump intensity q to depend conditionally on various state variables. Then, the transition probabilities for the interest rate following a Poisson–Gaussian process are written as (for $s > t$):

$$\begin{aligned} f[r(s)\,|\,r(t)] &= q\exp\left(\frac{-(r(s)-r(t)-k(\theta-r(t))\,\Delta t-\mu)^2}{2(v_t^2\,\Delta t+\gamma^2)}\right)\frac{1}{\sqrt{2\pi(v_t^2\Delta t+\gamma^2)}} \\ &\quad +(1-q)\exp\left(\frac{-(r(s)-r(t)-k(\theta-r(t))\,\Delta t)^2}{2v_t^2\,\Delta t}\right)\frac{1}{\sqrt{2\pi v_t^2\,\Delta t}}, \end{aligned} \tag{3.3}$$

[8] The Bernoulli approximation is achieved as follows: Define the indicator variable $Y_i = 1$ if a jump occurs, else $Y_i = 0$ for all $i \ldots N$, and where $\Delta t = T/N$, for the time series spanning T.

$$\begin{aligned} \Pr[Y_i = 0] &= 1 - h\Delta t + \mathrm{O}(\Delta t), \\ \Pr[Y_i = 1] &= h\Delta t + \mathrm{O}(\Delta t), \\ \Pr[Y_i > 1] &= \mathrm{O}(\Delta t). \end{aligned} \tag{3.1}$$

Let $M = \sum_{i=1}^{\mathrm{N}} Y_i$. M is distributed Binomial being the sum of independent Bernoulli variables. For x occurrences,

$$\Pr[M = x] = {}^{\mathrm{N}}C_x (hT/N)^x (1 - hT/N)^{N-x}, \quad \forall x,$$

$$\lim_{N\to\infty} \Pr[M = x] = \frac{\mathrm{e}^{-hT}(hT)^x}{x!}.$$

Therefore, the Bernoulli approximation converges to the appropriate Poisson density. Alternatively, the model used may be interpreted as a two-term truncation of a Poisson-weighted sum of normal distributions $[\sum_{n=0}^{\infty} [\mathrm{e}^{-h\,\mathrm{d}t}(h\,\mathrm{d}t)^n/n!\,f_n]$, where each normal distribution (f_n) is conditioned on the number of jumps $n, n = 0, 1 \ldots \infty$. This discussion parallels an alternative derivation in more generality in Ch3 of Merton (1992). Another reference on the Poisson approximation to the binomial distribution may be found in Feller (1951). Finally for option pricing, using the Poisson Limit Theorem, see Page and Sanders (1986).

where $q = h\,\Delta t + \mathrm{O}(\Delta t)$. This approximates the true Poisson–Gaussian density with a mixture of normal distributions. Estimation involves maximizing the function L, where

$$L = \prod_{t=1}^{T} f[r(t + \Delta t)\,|\,r(t)]$$

which may be written as:

$$\max_{\Omega=[k,\theta,v,\mu,\gamma^2,q]} \sum_{t=1}^{T} (\log(f[r(t + \Delta t)\,|\,r(t)])).$$

This discrete-time model parallels a model with a mixture of distributions. As in Ball and Torous (1983), and based on the technical specifications of the stochastic process. The technical regularity conditions stated in Cramer (1946, p. 500) are satisfied, and we obtain estimates that are consistent, unbiased, and efficient and attain the Cramer–Rao lower bound. In addition, this supports the application of maximum-likelihood and thence the likelihood ratio test to this model. The constraints are that the weights for each regime (jump vs. no-jump) add up to one, which is already imposed in the equation above, and that $0 \leqslant q \leqslant 1$, which is required at the time of estimation. Given the tight parallel of this model to that of mixture distributions, MLE is directly achieved as a solution to a system of first-order conditions $\partial \log L/\partial\Omega = 0$ (see Hamilton, 1994, Section 22.3, pp. 685–689). Estimation may be undertaken using gradient methods, or the E–M algorithm of Dempster et al. (1977).

Maximum likelihood estimation results are presented in Table 2. In order to compare different processes for the short rate, we estimated four nested models on the data set. The models estimated are (i) a pure-Gaussian model ($h = 0$), (ii) the Poisson–Gaussian model of Eq. (2.1), (iii) an ARCH–Poisson–Gaussian model, which consists of the Poisson–Gaussian model with the variance of the Gaussian component following an ARCH(1) process, [9] and

[9] This is done by modelling the conditional variance of the Gaussian process as follows:

$$v(s + \Delta t)^2 = a_0 + a_1[r(s) - \mathrm{E}(r(s)|r(t))]^2, \quad t < s$$

and estimating the parameters a_0, a_1. The recent work of Corradi (2000) clarifies the work of Nelson (1990) and shows that the continuous-time limit of ARCH type processes may be of two types, depending on the parameterization of the Euler approximation of the ARCH model taken to its continuous-time limit. In one case, we get a degenerate diffusion, i.e. a asset process and volatility process driven by one diffusion only. In the other case, we get a non-degenerate diffusion or stochastic volatility process, i.e. a two-dimensional process driven by a two-dimensional diffusion. The ARCH model in this paper falls in the former category.

Table 2
Basic Poisson–Gaussian estimation[a]

Parameter	Pure Gaussian	Poisson–Gaussian	ARCH–Poisson–Gaussian	ARCH–Gaussian
k	2.8832	0.8542	0.5771	1.2810
	3.68	2.26	2.02	4.66
θ	0.0576	0.0330	0.0346	0.0974
	10.91	2.57	2.50	9.78
v	0.0466	0.0173		
	111.01	24.01		
a_0			0.0001	0.0008
			17.65	66.48
a_1			127.0201	232.07
			13.92	29.85
μ		0.0004	0.0017	
		1.38	5.66	
γ		0.0058	0.0045	
		24.50	16.60	
q		0.2162	0.1564	
		17.91	13.14	
Log-likelihood	13938.13	14890.90	15197.67	14509.50

[a]We present results for the estimation of pure-Gaussian, Poisson–Gaussian, ARCH–Poisson–Gaussian and ARCH–Gaussian processes on daily data covering the period January 1988 to December 1997. The total number of observations is 2609. Estimation is carried out using maximum-likelihood incorporating the transition density function in Eq. (3.3). The discretized ARCH–Poisson–Gaussian process estimated is specified as follows:

$$\Delta r = k(\theta - r)\Delta t + v\,\Delta z + J(\mu, \gamma^2)\,\Delta\pi(q),$$

$$v_{t+\Delta t}^2 = a_0 + a_1[\Delta r_t - \mathrm{E}(\Delta r_t | r_{t-\Delta t})]^2.$$

The other processes are special cases of the one above. T-statistics are presented below the parameter estimates. The variable q, the probability of a jump in the interval Δt is analogous to the continuous time parameter h for jump arrival intensity, by the relation $q \approx h\Delta t$.

(iv) a pure ARCH–Gaussian model. This parallels to a large extent the analyses carried out by Jorion (1988) for the equity and foreign exchange markets.

Since the ARCH–Poisson–Gaussian model subsumes the other three models, likelihood ratio tests (LRT) may be applied to compare nested models. Comparison of nested log-likelihoods between the models in Table 2 reveals that the ARCH–Poisson–Gaussian model outperforms the rest. The likelihood ratio statistics are large and show ample evidence that the jump is well-founded as a modelling device. As would be expected the Poisson–Gaussian process fits the data significantly better than the pure-Gaussian one.

Whereas the Poisson–Gaussian and ARCH–Gaussian models are not nested, the likelihood for the Poisson–Gaussian model is greater, suggesting that Poisson–Gaussian processes provide a better fit than ARCH volatility models. Fig. 2 offers visual evidence for the better fit of the ARCH-jump model versus other specifications.

Application of the Akaike Information Criterion (not reported), where the likelihood is adjusted downwards by the number of parameters, provides evidence of this. The log-likelihood values are large and positive since the variance of conditional changes in interest rates is of the order Δt, and less than 1. In addition, paralleling the analysis depicted in Fig. 2, we computed distance estimates between the fitted probability density functions and the empirical distribution. Five measures of distance were used (RMSE, squared errors, goodness of fit, absolute differences and maximum entropy) and resulted in overwhelming support for the ARCH–Poisson–Gaussian model (these results are not reported, and are available on request). As raised in Hansen (1992), there may be an issue with the LRT test if the parameters for the jump intensity are not identified under the null. Hansen states that LRT is applicable when the likelihood surface is locally quadratic. If there is a nuisance parameter, such as the jump intensity, it may violate this condition and result in a flat likelihood function. In our case, we have a mixture of distributions satisfying required regularity conditions, and the likelihood function is well-behaved. In order to check this, we implemented a version of the idea proposed in Hansen's paper, where he suggests using a grid approach. In Fig. 3, we present the plot of the likelihood function for varying values of the nuisance parameter. The numerical analysis and plot confirms that the function is quadratic, and also that it is not flat.

A comparison of the pure-Gaussian model and the Poisson–Gaussian model reveals a sharp drop in Gaussian volatility (v) when jumps are introduced into a pure-Gaussian model, suggesting that jumps account for a substantial component of volatility. For example, in Table 2, the Gaussian volatility drops to one-third its prior level. The unconditional mean of the interest rate under the discretized process is given by $\theta + h\mu = \theta + q\mu/\Delta$, and computations using the values in Table 2 arrive at a value of 0.0557 or 5.57%, once again close to the mean value in Table 1.

In the Poisson–Gaussian model (Table 2) we find that $q = 0.2162$, which under our Bernoulli model is simply the probability of a jump on any day. Thus, we find that jumps occur once every five days over our sample period. In contrast, the ARCH–Poisson–Gaussian model provides a jump probability of only 0.1564, evidence of the fact that stochastic volatility will account for some of the fatness in the tails of the distribution in preference to jumps. While it is clear that pure-Gaussian models do not capture the features of the data, Poisson–Gaussian and ARCH–Gaussian models as well fall short of the efficacy of the ARCH–Poisson–Gaussian model. This suggests that future

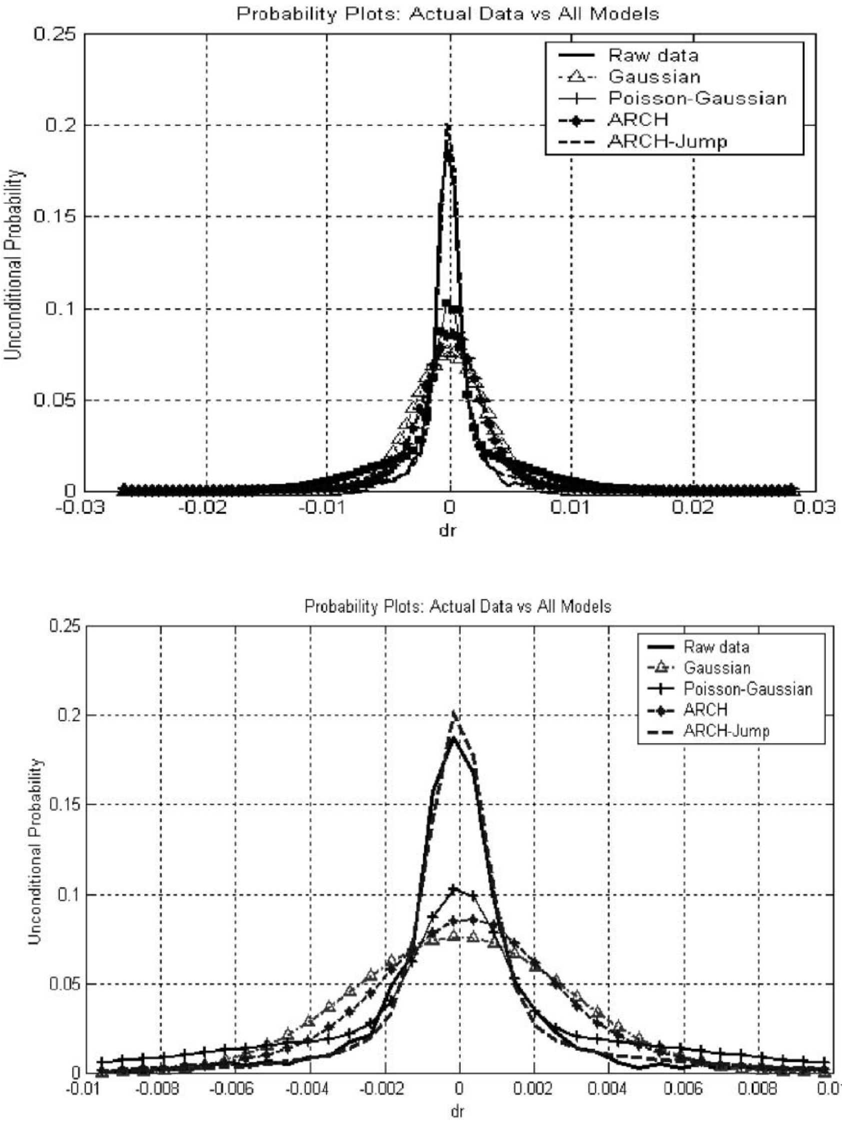
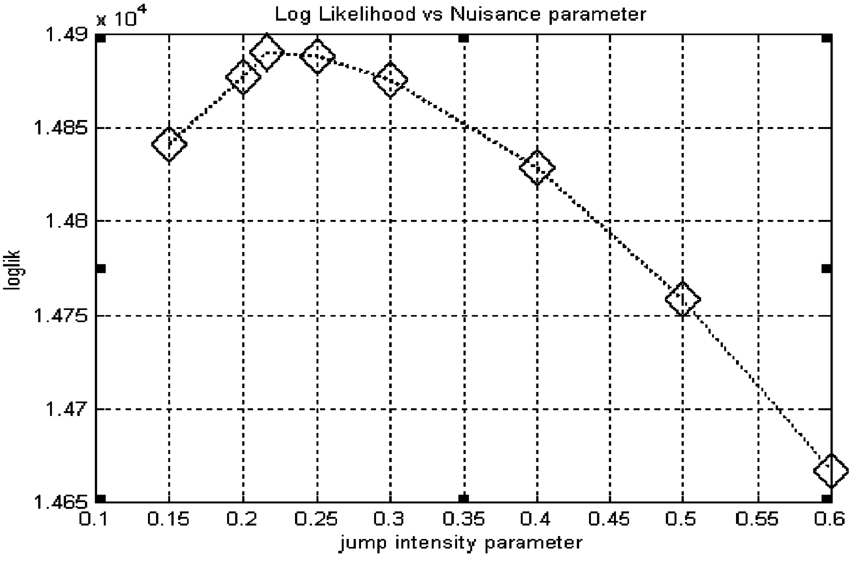


Fig. 2. The unconditional probability density function from the raw data and the plots from the best-fitted models of each type is presented. The upper panel presents the full distribution over the range $(-0.03, 0.03)$, and leptokurtosis is evident. Only the ARCH-jump model is able to match the histogram of the raw data. The lower panel zooms in on the same distribution where dr lies in the interval $(-0.01, +0.01)$, the closer plot clearly brings out the good fit from the ARCH-jump model compared to the other models.



Intensity	Loglik	LR
0.1	14841.23	99.34
0.15	14877.60	26.59
0.2	14890.21	1.38
0.2162	14890.90	0.00
0.25	14888.22	5.35
0.3	14875.94	29.92
0.4	14828.88	124.04
0.5	14758.46	264.88
0.6	14667.14	447.51

Fig. 3. Diagnostic results to assess the impact of the jump intensity parameter. As per Hansen (1992), the likelihood function will be flat in the nuisance parameter, in this case, the jump intensity (lying between 0 and 1). To investigate this we plotted the likelihood function for varying values of the nuisance parameter. The table accompanying the plot also shows, by means of a likelihood-ratio (LR) stat, that the likelihood varies significantly as we move away from the optimal jump intensity. Our model closely parallels the one described in Hamilton (1994, pp. 685–689) for mixture distributions, and derives its nice properties from the analysis there.

theoretical work be driven in the direction of a combined ARCH–Poisson–Gaussian model. [10]

[10] While the models here retain the assumption of Markovian processes, they relax the assumption of diffusions only, by admitting jumps. See Ait-Sahalia (1998) for a technique to examine the presence of discontinuity in continuous-time Markov diffusions.

We extend the basic Poisson–Gaussian model by allowing the jump mean parameter to vary. It is likely that the jump size distribution is positively skewed at low levels of r and negatively skewed at high levels of r. This can be modeled by allowing the mean of the jump size to depend on the level of r. For example, we may use the following specification for the jump mean: $\mu_t = \alpha_0 + \alpha_1(\theta - r_t)$. When $\alpha_1 > 0$, we obtain sharp mean reversion of the short rate through the jump component of the process. This type of reversion may be driven more by information surprises or by an overreaction response, and less by macroeconomic cycles. Another approach to the handling of this model is via a regime-switching model of interest rates, as considered in Gray (1996) and Ang and Bekaert (1998). While this paper is enhanced with jump processes, the differential rates of mean reversion across regimes in Gray's paper, may be indicative of the jump phenomenon being explored in this paper. Full consideration of this type of model is presented in Section 4.4.

Table 3 reports the results of the time-varying mean reverting model when jumps inject mean reversion. The mean reversion in the process is now attributable to both the drift term and the jump term. Since jump arrivals are uncertain, the rate of mean reversion is now time-varying, and the drift in the interest rate becomes stochastic. Ait-Sahalia (1996a,b) and Stanton (1997) demonstrate that the drift term displays non-linear behavior, which may be partially explained if jumps inject 'extra' mean reversion at interest rates far away from the long run mean of the short rate. In fact these papers find that the mean reversion pull is far stronger when the interest rate lies outside the range 4%–17%, which is consistent with the phenomenon suggested here. We extend our empirical model to estimate the parameters (α_0, α_1). We estimated the Poisson–Gaussian and ARCH–Poisson–Gaussian model with time-varying jump means (Table 3). The T-statistic for α_1 is significant for the jump model indicating that the mean of the jump process is time-varying. However, when an ARCH effect is added to the model, the time-varying drift coefficient becomes insignificant. The joint evidence of these two models appears to suggest that different specifications of the volatility and jump may result in a linear drift model. We explore this issue in greater detail in a later subsection.

3.2. Diagnostics

It is instructive to examine the empirical moments over different data intervals (see Das and Sundaram, 1999). Define the time interval between observations in the data as T. From Section 2.3, the conditional variance of the jump-diffusion process is:

$$\mu_2 - \mu_1^2 = \frac{v^2 + h\mathrm{E}[J^2]}{2k}(1 - \mathrm{e}^{-2kT}).$$

Table 3
Estimation of the time varying jump means model[a]

Model	Jump-diffusion		ARCH-jump	
Parameter	Estimate	T-statistic	Estimate	T-statistic
k	0.6336	1.64	0.5023	1.72
θ	0.0233	1.04	0.0304	1.69
v	0.0173	24.04		
a_0			0.0001	17.65
a_1			126.6914	13.90
q	0.2163	17.91	0.1567	13.14
α_0	0.0018	1.46	0.0022	3.13
α_1	0.0414	3.04	0.0202	1.44
γ	0.0057	23.16	0.0045	16.03
Log-likelihood	14895.65		15198.82	
Log-likelihood for constant μ	14890.90		15197.67	
P-value for $\chi^2(1)=$	0.0021		0.1294	

[a]We present results for the estimation of the ARCH–Poisson–Gaussian model allowing for time variation in the mean of the jump size. This enables assessment of the mean reversion effects of the jump process. Estimation is carried out using maximum-likelihood incorporating the transition density function in Eq. (3.3). The process estimated is specified in the following equations:

$$\Delta r = k(\theta - r)\,\Delta t + v\,\Delta z + J(\mu, \gamma^2)\,\Delta\pi(q),$$

$$v_{t+\Delta t}^2 = a_0 + a_1[\Delta r_t - \mathrm{E}(\Delta r_t | r_{t-\Delta t})]^2,$$

$$\mu_t = \alpha_0 + \alpha_1(\theta - r_t).$$

T-statistics are presented below the parameter estimates. The χ^2 statistic is computed for twice the difference between the mean reverting jump model and the constant mean jump model (where $\alpha_0 = \mu$ and $\alpha_1 = 0$). The degrees of freedom used is one, being the difference in the number of parameters between the two models. The variable q, the probability of a jump in the interval Δt is analogous to the continuous time parameter h for jump arrival intensity, by the relation $q \approx h\Delta t$.

The skewness is

$$\text{Skewness} = \frac{\mathrm{E}(J - \mu_1)^3}{(\mu_2 - \mu_1^2)^{3/2}}$$

$$= \frac{2\sqrt{2k}\mathrm{e}^{-kT}(1 + \mathrm{e}^{kT} + \mathrm{e}^{2kT})h\mathrm{E}(J^3)}{3(1 + \mathrm{e}^{kT})(v^2 + h\mathrm{E}(J^2))\sqrt{(1 - \mathrm{e}^{-2kT})(v^2 + h\mathrm{E}(J^2))}}.$$

If $h=0$, then the skewness is zero. The kurtosis of the process is:

$$\text{Kurtosis} = \frac{\mathrm{E}(J-\mu_1)^4}{(\mu_2-\mu_1^2)^2}$$
$$= \frac{(\mathrm{e}^{2kT}-1)(3h^2\mathrm{E}(J^2)^2+6hv^2\mathrm{E}(J^2)+3v^4)+kh\mathrm{E}(J^4)(\mathrm{e}^{2kT}+1)}{(\mathrm{e}^{2kT}-1)(v^2+h\mathrm{E}(J^2))^2}.$$

When $h=0$, i.e. no jumps, the kurtosis is 3, which is the normal level. The conditional kurtosis declines monotonically as T increases.

As a first check we compute the moments of the conditional distribution of interest rates using the estimated parameters for the jump-diffusion model in Table 2. Given the estimated values for the jump distribution (μ, γ^2), we can compute the following values: $\mathrm{E}(J)=\mu$, $\mathrm{E}(J^2)=\mu^2+\gamma^2$, $\mathrm{E}(J^3)=\mu^3+3\mu\gamma^2$, and $\mathrm{E}(J^4)=\mu^4+6\mu^2\gamma^2+3\gamma^4$. Since our data is daily, the horizon T is $1/262=3.8462\times10^{-3}$, given the number of trading days in a year. In order to make a rough comparison, the moments of the change in interest rates ($\mathrm{d}r$) in Table 1 will correspond to the computed moments at $T=1/262$. In fact they correspond well. The values are (values from Table 1 are in brackets): standard deviation = 0.0029 (0.0029), skewness = 0.3553 (0.3950), and kurtosis = 13.36 (19.86).

To further assess the ability of the different models to match the features of the data, we carry out some diagnostics on the models using the parameter estimates of Table 2. What we shall see is that the ARCH-jump model provides a very good fit to the data, whereas the other models are found lacking. We undertake this analysis in two ways. First, a simple comparison of log-likelihoods reveals that the ARCH-jump-diffusion model vastly outperforms the other three nested models, i.e. the pure-diffusion, pure-jump and ARCH models. Second, we assess how well the probability distributions of the estimated models match that of the data.

In order to do so, we compare the unconditional probability density functions of the changes in interest rates for our estimated models with that of the unconditional distribution obtained from the data. Fig. 2 presents the results of this analysis. In all the figures, the density from raw data (dashed line) is drawn using a smoothed histogram and displays extremely high peakedness and fat tails. The shorter, smoother distribution (solid line) is based on the theoretically estimated model. Fig. 2 shows that the best-fit Gaussian model is substantially different from that of the data which is far more leptokurtic. The jump model improves the fit somewhat, but is still quite different, as is also the case for the ARCH model. There is insufficient leptokurtosis generated in either model. However, the ARCH-jump model in Fig. 2 shows a very good fit, pointing to the fact that both jumps and conditional heteroskedasticity are predicated in the search for the best specification.

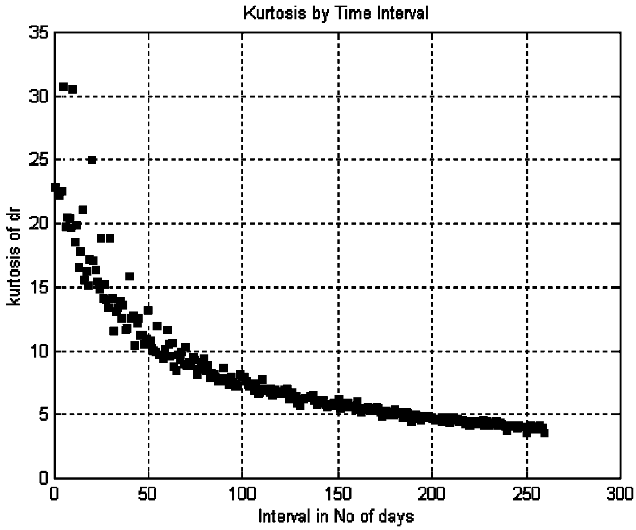


Fig. 4. The plot of the term structure of kurtosis from the jump-diffusion interest rate model in the paper. As described in Section 3.3, the kurtosis of future interest rates is computed using the data intervaled at various maturities leading to a depiction of the 'term-structure' of the fourth moment. The kurtosis here pertains to the changes in interest rates, and is therefore very high at short maturities, and then decays rapidly to the normal asymptote of 3.

3.3. Term structure of kurtosis

There is substantial evidence of kurtosis in the data, as seen from Table 1 and Fig. 2. The different models imply varied behavior of kurtosis as we vary the time interval. We may understand this behavior well by examining the term structure of kurtosis.

We use the higher moments to understand the differences between the diffusion-based class of models and the jump-diffusion class. As the time interval for sampling the process, i.e. T varies, the conditional skewness and kurtosis also varies. For a stochastic volatility diffusion model, skewness and kurtosis increase with sampling interval for a while and then drop back to Gaussian levels when the sampling interval becomes long. In the case of a jump-diffusion model skewness and kurtosis start at a very high level and then decline monotonically to asymptote with Gaussian levels. These issues are discussed in detail in Das and Sundaram (1999).

The plot in Fig. 4 depicts the kurtosis for interest rate changes where the time interval between observations varies from 1 day to 260 days. The plot has been generated by intervaling the data for n days, where $n = 1, 2, \ldots, 260$. When $n > 1$, the data set yields more than one intervaled time series; for example, when $n = 2$, we have two series, each 2 days apart. The reported

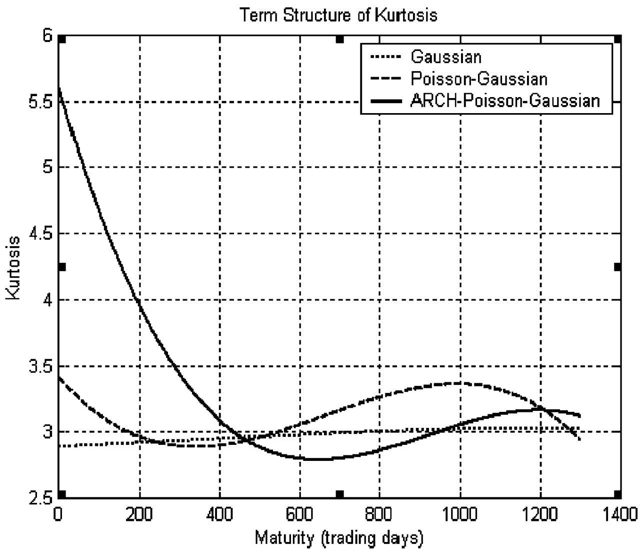


Fig. 5. The plot of the term-structure of kurtosis from the three estimated models in the paper: (i) the pure Gaussian model, (ii) the Poisson–Gaussian model, and (iii) the ARCH–Poisson–Gaussian model. The plot for each model was generated using 500 sample-path simulations, and then computing the kurtosis for each maturity from 500 observations each day. We plot the kurtosis of the interest rate level (versus that of changes in the interest rate).

kurtosis is the average of the kurtosis of each series. This eliminates to some extent any day-of-week effects that might affect the graph. The monotonic decline in kurtosis is unmistakable, and confirms two aspects of term-structure models already identified previously in the empirical section: (i) that jumps exist, since the declining kurtosis plot would not arise from a pure-diffusion model alone, unless it were mixed with a jump process, and (ii) in the case of a mixed jump-diffusion model, a declining plot would only arise if jumps constituted a substantial component of the variation in the interest rate sample path. This, as we have seen from the results in Table 2, is certainly the case.

Fig. 5 explores the kurtosis of r for horizons out to 5 years. The term structure of kurtosis was generated using a simulation methodology for the three processes of interest: (i) the pure Gaussian model, (ii) the Poisson–Gaussian model and (iii) the ARCH–Poisson–Gaussian model. For each model I generated 500 sample paths, on a daily frequency, using the estimated parameters from Table 2. From these paths the kurtosis is computed for horizons from 1 day to 1250 trading days. As expected the pure-Gaussian model should evidence no excess kurtosis in theory, and the graph is almost flat. The Poisson–Gaussian model will theoretically show a monotonically declining kurtosis, and this is also seen. Finally, an ARCH–Poisson–Gaussian model is capable

Table 4
Assessment of the discrete estimator[a]

Statistic	Parameter					
	k	θ	v	μ	γ	q
True parameter	0.8542	0.0330	0.0173	0.0004	0.0058	0.2162
Estimated parameter	0.9589	0.0342	0.0173	0.0004	0.0058	0.2162
Standard error	0.2587	0.0017	0.0004	0.0003	0.0002	0.0126
T-statistic	0.4048	0.1035	0.0215	0.0114	0.0563	0.0025
Minimum	0.3760	−0.0173	0.0162	−0.0004	0.0052	0.1860
Maximum	1.8828	0.0699	0.0187	0.0013	0.0065	0.2538

[a]This table presents results for a simulation evaluation of the estimator. Using the true values of the parameters for the jump-diffusion model, we simulated 500 sample paths of length 2609 daily observations each. For each sample path we undertook discrete MLE estimation. The following table presents summary statistics of the 500 simulated estimations. We computed the mean and standard error for the parameters estimated, and computed t-statistics for the difference in the simulated parameter estimate versus the true parameter. None of these t-statistics were significant.

of demonstrating myriad behavior, since the jump portion results in declining kurtosis, while the ARCH portion may cause an increase in kurtosis, with an eventual decline. Also, the overall levels of kurtosis will be higher since the tails are fatter in the ARCH–Poisson–Gaussian model. All these aspects are confirmed in the graph, complementing the analysis in Fig. 2. Also note that the term structure of kurtosis exhibits the same snake-like behavior seen in the term structure of interest rate volatility in the paper by Piazzesi (1998).

3.4. Assessment of estimation accuracy

A simulation exercise was undertaken to determine how well the estimator performed. The parameters for the Poisson–Gaussian model in Table 2 were used to generate 500 sample paths from the jump-diffusion model beginning with an initial interest rate of 7.1%, which is exactly as it was in the sample. Each sample path was for a daily time interval, and contained 2609 observations. For each generated sample path, estimation was undertaken by discrete maximum-likelihood. The estimates are provided in Table 4. We report the true parameter values, along with those from the estimation. The small standard errors suggest that the estimation procedure is very accurate.[11]

[11] Other simulation studies that have been undertaken are those by Chapman and Pearson (2000) and Pritsker (1998).

4. Applications

In this section we shall employ the model to examine various phenomena in the bond markets via the lens of the model. Our jump model is facile in permitting many different analyses. Jumps may be used to find day-of-week effects. We undertake an extensive analysis of Fed activity, and the model is found to account for some of the non-linearity of the drift term when the short rate is modeled purely as a diffusion process. We use jumps as a way of testing for bond market overreaction, and then extend the model to regime-shifting in the state variables. We discuss these applications one by one.

4.1. Day of the week effects

In this section we examine whether jumps are more likely to occur on specific days of the week, by introducing a modification to make the arrival intensity of jumps depend on the day of the week. There are several reasons which make jumps more likely on some days of the week rather than others. For example, jumps would be more likely on Mondays since the release of pent up information over the weekend may drive up the possibility of a large change in interest rates. Likewise, option expiry may inject jumps into the behavior of interest rates, and this would be more likely on Wednesdays and Thursdays. Jumps may also occur on Fridays when last minute trading may create excess volatility.

By using dummy variables for each day of the week, we assume a linear model for the arrival intensity of jumps in the short rate of interest:

$$q_t = \lambda_0 + \lambda_1 d_1 + \lambda_2 d_2 + \lambda_3 d_3 + \lambda_4 d_4,$$

where λ_0 is the arrival probability of a jump if the day is Friday, and $\lambda_i,\ i = 1–4$ is the incremental arrival intensity of jumps over Friday's level when the day of the week is Monday, Tuesday, Wednesday and Thursday, respectively. $d_i,\ i = 1–4$ are dummy data variables indicating the day of the week for Monday, Tuesday, Wednesday and Thursday, respectively. Estimation was conducted over the two models containing jumps, i.e. (i) the jump-diffusion model and (ii) the ARCH-jump-diffusion model. The results of the estimation are presented in Table 5.

Intuitive results emanate from this analysis. There is little evidence of skewness ($\mu \simeq 0$), but kurtosis exists ($\gamma > 0$). The jump tends to be of the order of 50 basis points. The likelihood of jumps is highest on Fridays, but jumps are also likely on Wednesdays and Thursdays, when information from options expiry is released. This lends credence to the proposition that jumps are caused by large bursts of information being released into the market.

Table 5
Jump estimation with day of the week effects[a]

Parameter	Estimate	T-statistic
k	0.7960	2.09
θ	0.0259	1.60
v	0.0171	24.38
λ_0	0.1222	6.17
λ_1	0.0413	1.34
λ_2	0.0147	0.54
λ_3	0.2523	6.85
λ_4	0.1777	5.46
μ	0.0004	1.52
γ	0.0057	24.94
Log-likelihood	14932.74	

[a]The table presents results of the estimation of a jump-diffusion model when the jump arrival intensity is assumed to be affected by the day of the week. The jump intensity follows a linear model

$$q_t = \lambda_0 + \lambda_1 d_1 + \lambda_2 d_2 + \lambda_3 d_3 + \lambda_4 d_4,$$

where d_i, $i = 1$–4 are dummy variables for Monday, Tuesday, Wednesday and Thursday, respectively.

4.2. Federal reserve activity

Jumps may arise from intervention by the Federal Reserve in the bond markets. The Federal Open Market Committee (FOMC) meets periodically, and informs their open market desk of the range they wish to establish for the Fed Funds rate. Short rates tend to track this rate rather closely. It is possible that these meetings form an important information event. If so, a model that accounts for this will prove to be superior for traders. In this section, we enhance our jump model by making the jump intensity depend on the FOMC meeting. By examining the impact of the meeting on the jump probability we can ascertain whether the meeting is a significant information event.

This section complements the work of Hamilton (1996), Hamilton and Jorda (1977) and Demiralp and Jorda (1999), as well as the earlier work of Balduzzi et al. (1997, 1998a, b). These papers use various econometric specifications to model the impact of Federal Reserve activity on the yield curve. In this paper, the specific modelling approach is to use a model with jumps, as is also undertaken in the work of Piazzesi (1998, 1999).

The FOMC meets eight times each year. There are two types of meetings of the FOMC: one-day meetings and two-day meetings. There are usually 2 two-day meetings and 6 one-day meetings every year. Our sample over the ten-year period consists of 58 one-day meetings and 22 two-day meetings. In total there were 80 meetings, i.e. one every 6–7 weeks. The first and fourth meetings every calendar year are two-day events. They begin at 2:30 pm on the first day, continuing at 9:00 am the following day. The one-day meetings always begin at 9:00 am. All meetings begin on Tuesdays.

At these meetings, the FOMC examines information about the economy and decides on whether to undertake open market operations in the dollar or other currencies. They also determine the level of short-term rates. The usual issues relating to the economic outlook are considered: consumer spending, non-farm payroll, industrial production, retail sales, real business fixed investment, nominal deficit, consumer price inflation, currency rates, money supply (M2, M3), and housing activity. At the two-day meetings additional policy directives are issued. In particular, these relate to domestic open market operations, authorization of foreign bank limits for foreign currency operations, foreign currency directives, and procedural instructions with reference to foreign currency operations. We find that the two-day meetings appear to have a greater information impact than one-day meetings.

In addition to foreign currency directives, the Fed also undertakes other distinct activity at the two-day meetings. By (the Humphrey–Hawkins) law, the Fed must report to Congress twice a year on monetary policy, i.e. in February and July. The two-day meetings are the setting for the discussions on monetary policy as well. The FOMC thus votes on the range of growth rates of M2, M3 and the debt levels it expects to see. Thus, two-day meetings tend to evidence more forward-looking discussions than usually occur at one-day meetings. However, these votes are not announced immediately, and only get reported in minutes two weeks after the meeting. Thus, it is not clear that this activity of the Fed in any way forms an information event. However, we do find that the two-day meetings seem to impact parameter estimation, in contrast to the one-day meetings.

To begin, we carry out a few simple regressions to ascertain if the volatility of interest rates is in any way related to information released at FOMC meetings. This is done by regressing the squared change in interest rates on interest rate level and a dummy variable for the FOMC meeting. The regression equation is as follows:

$$[r_{t+1} - r_t]^2 = a + br_t + cf_{t+1} + e_{t+1},$$

where f_t is the dummy variable indicating the FOMC meeting. It may take four different forms as described in Table 6 below. Since some of the meetings last 2 days, combinations are possible. First, we assign a dummy variable which is the first day of all meetings. As can be seen, this has little impact on volatility, and hence provides evidence of no unexpected information. A similar result holds when we examine only one day meetings. However, when we set the dummy variable to be the first day of a two-day meeting,

Table 6
FOMC meeting impact: linear regressions[a]

Dummy variable (f_t)	a	b	c	R^2
1st day all meetings	0.0373	0.0078	0.0441	0.0019
	1.55	1.98	0.99	
1st day, 1 day meetings	0.0388	0.0078	−0.0120	0.0016
	1.61	1.99	−0.27	
1st day, 2 day meetings	0.0377	0.0075	0.3178	0.0070
	1.57	1.91	3.79	
2nd day, 1 day meetings	0.0381	0.0078	0.0723	0.0018
	1.58	1.98	0.86	

[a]We examine via simple regressions whether the FOMC meeting results in a information surprise. The regression is

$$[r_{t+1} - r_t]^2 = a + br_t + cf_{t+1} + e_{t+1},$$

where f_t is the dummy variable for the FOMC meeting. T-statistics are presented below the parameter estimates.

the coefficient comes in strongly positive. This indicates that there may be a significant information release on the first day of the 2-day FOMC meetings. We also examined whether the information impact occurred on the second day of the two-day meeting and found little effect. Thus, if there is an information effect, it occurs on the first day of the two-day meeting. Table 6 summarizes the regression results.

This informal regression proxies for the possible impact of the FOMC meeting on interest rate changes. We now turn to the examination of whether the probability of a jump is linked in any way to the FOMC meetings. We achieve this using a modification of our Poisson–Gaussian estimation model depicted in Eq. (3.3). In the estimation Eq. (3.3), we specify that the arrival probability of a jump, denoted by the parameter q, be a function of the Fed meetings (f_t). It is possible that jumps in the interest rate are caused by Fed actions, and then the information on meetings would determine the probability of a jump taking place. Thus we specify

$$q_t = \lambda_0 + \lambda_1 f_{1t} + \lambda_2 f_{2t}.$$

The equation above accommodates a base level of jump probability λ_0, augmented by Fed dependent attributes, λ_1, λ_2, for one-day and two-day FOMC meetings, respectively. For the ARCH-diffusion model, we investigate whether the Fed meetings have an impact on conditional volatility by specifying the ARCH equation with an additional coefficients $a_{1\,\mathrm{day}}, a_{2\,\mathrm{day}}$ on the Fed event, i.e. the variance will be $a_0 + a_1\varepsilon_t^2 + a_{1\,\mathrm{day}} f_{1t} + a_{2\,\mathrm{day}} f_{2t}$. First, we examine the one-day meetings only (results not reported). The one-day meetings appear to have very little impact on the usual levels of jump probability, as seen in the jump-diffusion model. The parameter λ_1 is not significant. And in fact,

Table 7
FOMC meeting impact: one-day and two-day meetings[a]

Model	ARCH-diffusion		Jump-diffusion	
Parameter	Estimate	T-statistic	Estimate	T-statistic
k	1.2677	4.63	0.8660	2.26
θ	0.0973	9.72	0.0340	2.64
v			0.0173	24.04
a_0	0.0008	65.66		
a_1	232.9507	29.82		
$a_{1\text{day}}$	−0.0003	−4.09		
$a_{2\text{day}}$	0.0009	1.81		
λ_0			0.2114	17.39
$\lambda_{1\text{day}}$			0.0832	1.21
$\lambda_{2\text{day}}$			0.3422	2.11
μ			0.0003	1.34
γ			0.0057	24.48
Log-likelihood	14513.39		14894.69	

[a]We examine via ARCH and jump models whether the FOMC meeting results in a information surprise. The jump model is extended by $q_t = \lambda_0 + \lambda_{1\text{day}} f_{1t} + \lambda_{2\text{day}} f_{2t}$ where f_{1t}, f_{2t} are the dummy variables for the FOMC meetings. The ARCH model is written as $a_0 + a_1 \varepsilon_t + a_{1\text{day}} f_{1t} + a_{2\text{day}} f_{2t}$.

the ARCH model evidences a decrease in volatility when a one-day FOMC meeting takes place. However, when we used the two-day meetings only, the information impact of this dummy variable proves to be significant, i.e. it increases the probability of a jump. This probability more than doubles in magnitude. Finally, we put both one-day and two-day meetings together in one model and ascertain the results in Table 7. The two-day meetings result in a sharp increase in the possibility of a jump. The one-day meetings in fact seem to predicate a reduction in conditional volatility. One might speculate that the two-day meetings do result in information surprises, whereas the one-day meetings confirm the market's forecasts. Recent work by Piazzesi (1998, 1999) supports these results, with the finding that jump-diffusion models may be used to capture target-rate moves on FOMC meeting days, via the use of target rates as an observable factor in a three-latent-factor framework. Piazzesi also finds that yield volatility rises on FOMC meeting days.

4.3. The pervasiveness of the non-linear drift

In recent papers, Ait-Sahalia (1996a, b), Conley et al. (1997), Stanton (1997) have found the drift of the short rate to be non-linear in the lagged interest rate. Extending this specification by introducing jumps may render the drift linear in interest rates. We explore this aspect in this section. More recent papers have addressed this issue as well, see for example, Bodoukh

Table 8
Model estimation with non-linear drift[a]

Model	Pure-diffusion		Jump-diffusion		ARCH-jump	
Parameter	Estimate	T-statistic	Estimate	T-statistic	Estimate	T-statistic
k	−81.6448	−4.67	−25.6544	−2.08	−20.0436	−2.09
θ	0.0546	36.86	0.0580	18.88	0.0554	23.33
α_2	−475.9293	−4.90	−142.9902	−2.02	−115.6292	−2.10
α_3	0.0769	4.57	0.0264	2.38	0.0189	2.20
v	0.0465	108.47	0.0173	23.96		
a_0					0.0001	17.65
a_1					127.0719	13.95
$q = h\,\mathrm{d}t$			0.2162	17.90	0.1553	13.07
μ			0.0004	1.43	0.0017	5.69
γ			0.0058	24.46	0.0045	16.55
Log-likelihood	13944.11		14894.29		15200.17	

[a]This table presents the results of the estimation model where the drift term is non-linear. The model specification is as follows:

$$\Delta r = [k(\theta - r) + \alpha_2 r^2 + \alpha_3/r]\,\Delta t + v\,\Delta z + J(\mu, \gamma^2)\,\Delta\pi(q),$$

$$v^2_{t+\Delta t} = a_0 + a_1[\Delta r_t - \mathrm{E}(\Delta r_t | r_{t-\Delta t})]^2.$$

et al. (1998), Ang and Bekaert (1998), Ahn and Gao (1999), and Duffee (1999).

We estimate four models allowing for non-linear drift terms: (i) a pure-diffusion model, (ii) a jump-diffusion model, (iii) an ARCH-diffusion model and (iv) an ARCH-jump-diffusion model. The ARCH-diffusion model failed to converge. The general econometric specification is as follows:

$$\Delta r_t = [k(\theta - r_t) + \alpha_2 r_t^2 + \alpha_3/r_t]\,\Delta t + v_t\,\Delta z_t + J(\mu, \gamma^2)\,\Delta\pi(h),$$

$$v_{t+\Delta t} = a_0 + a_1[r_t - \mathrm{E}(r_t | r_{t-\Delta t})]^2.$$

The critical parameters are (α_2, α_3). They examine whether the drift is a function of squared interest rates or inversely related to interest rate levels. If any of these parameters is significantly different from zero, it means that the drift term is non-linear. The results are presented in Table 8. An extended specification is also used and is presented in Table 9. Fig. 6 offers a graphical exposition too. It is evident from the table and figure that the introduction of the jump does diminish the size of the non-linear coefficients (α_2, α_3). There is also a reduction in the level of significance. In fact the non-linearity parameters are significant at the 95% level but not at the 99% level once the jump model is introduced. Hence, it is possible that the jumps do make the model linear in drift.

Table 9
Estimation of the time varying jump means model with non-linear drift[a]

Model	Jump-diffusion	
Parameter	Estimate	T-statistic
k	−26.9835	−2.18
θ	0.0577	20.27
α_2	−149.6366	−2.11
α_3	0.0272	2.45
v	0.0173	23.98
q	0.2163	17.90
α_0	0.0004	1.23
α_1	0.0419	3.07
γ	0.0057	23.15
Log-likelihood	14899.13	

[a]We present results for the estimation of the Poisson–Gaussian model allowing for time variation in the mean of the jump size when the drift term is non-linear. This enables assessment of the mean reversion effects of the jump process, and its impact on the drift. Estimation is carried out using maximum-likelihood incorporating the transition density function in Eq. (3.3). The process estimated is specified in the following equations:

$$\Delta r = [k(\theta - r) + \alpha_2 r^2 + \alpha_3/r]\,\Delta t + v\,\Delta z + J(\mu, \gamma^2)\,\Delta\pi(q),$$

$$\mu_t = \alpha_0 + \alpha_1(\theta - r_t).$$

T-statistics are presented below the parameter estimates. The variable q, the probability of a jump in the interval Δt is analogous to the continuous time parameter h for jump arrival intensity, by the relation $q \approx h\,\Delta t$.

4.4. Overreaction in the bond markets

Examining the time series of the Fed funds rate shows that the market often overreacts, i.e. large moves in the interest rate are followed by speedy reversals. This was pointed out in the paper by Hamilton (1996), where he undertook a set of tests to examine if the Fed funds rate was a martingale.[12] He found that the martingale property failed to hold. The jump model in this paper offers an alternative approach to examining this issue.[13] The existence of overreaction would mean that the direction of the interest rate would be predictable after large moves. If overreaction exists, then the probability of a

[12] Hamilton also regressed the changes in interest rates on a constant and a dummy for the two largest changes in interest rates (p. 32), finding an R^2 of 0.41 (for the period 1984–1990). He concluded that outliers were an important aspect of the data and should be modelled. This paper offers jump-diffusion models as the mechanism by which to account for this distributional property. Replicating his regression for the period 1988–1997 delivers an R^2 value of only 0.07. Thus the impact of outliers is less in the sample period used here.

[13] I am grateful to David Bates for suggestions that led to the ideas here.

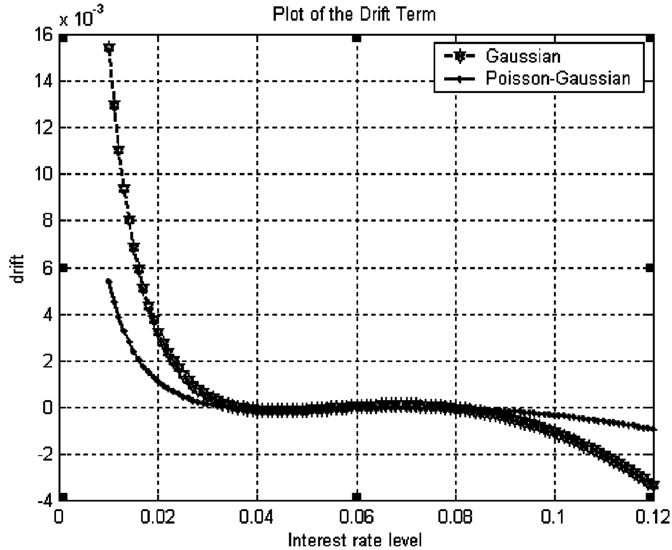


Fig. 6. The drift terms from the Gaussian model and the jump-diffusion models estimated in Table 10. A reduction in the non-linearity of the drift is evidenced.

jump in a direction opposite to that of the previous large movement in rates would outweigh the probability of a jump in the same direction as the prior move. To test this, we modify the jump model by making the jump intensity a function of the product of the current and prior change in interest rates. Thus,

$$
\begin{aligned}
q_t &= q_0 + q_1 \max[0, (r_t - r_{t-\Delta t})(r_{t-\Delta t} - r_{t-2\Delta t})] \\
&\quad + q_2 \min[0, (r_t - r_{t-\Delta t})(r_{t-\Delta t} - r_{t-2\Delta t})] \\
&= q_0 + q_1 R_t^+ + q_2 R_t^-,
\end{aligned}
$$

where the variables $[R_t^+ = (r_t - r_{t-\Delta t})(r_{t-\Delta t} - r_{t-2\Delta t}),\ R_t^- = (r_t - r_{t-\Delta t})(r_{t-\Delta t} - r_{t-2\Delta t})]$ capture the asymmetry and magnitude of continuations and reversals in the data. $R_t^+ > 0$ is the continuation coefficient, and $R_t^- < 0$ is the reversal component. The results from the estimation of this model are provided in Table 10. The coefficients q_1, q_2 are both significant indicating that they impact jump intensity strongly. The average values of R^+ and R^- are 0.0145 and -0.039, respectively. The average jump intensity q in this model is 0.3. Of this, reversals account for approximately 75% of the jump intensity. Thus, there is strong evidence in favor of market overreaction. This also indicates that only 25% of the jumps actually persist, and result in a permanent shift in the level of interest rates.

Table 10
Jump intensity as a function of reversals and continuations[a]

Parameter	k	θ	v	q_0	q_1	q_2	μ	γ
Estimate	0.5375	0.0359	0.0176	0.0034	4.7879	−5.8003	0.0001	0.0063
T-statistic	1.52	1.99	29.98	1.12	61.87	−105.66	0.31	42.35
Log-likelihood	15496.07							

[a]We present results for the estimation of the Poisson–Gaussian model allowing for time variation in the jump intensity as a function of continuations and reversals in the time series. Estimation is carried out using maximum-likelihood incorporating the transition density function in Eq. (3.3). The process estimated is specified in the following equations:

$$\Delta r = k(\theta - r)\,\Delta t + v\,\Delta z + J(\mu, \gamma^2)\,\Delta\pi(q_t),$$

$$q_t = q_0 + q_1 R_t^+ + q_2 R_t^-.$$

T-statistics are presented below the parameter estimates.

This finding leads to an extension of the model to account for the dichotomous behavior of the Fed funds rate. As pointed out in Balduzzi et al. (1998a), the Fed funds rate has two components: persistent changes in the target rate, and frequent departures from the target rate, which rapidly correct to the target. The former may be modeled as regime shifts in the central tendency of the short rate of interest, as in the paper by Balduzzi et al. (1997). The latter feature comprises bond market overreaction. We now develop an extension of our model to accommodate regime changes in the target rate. This was modeled by Gray (1996), and Naik and Lee (1993) and the specification here undertakes a similar exercise with shifting regimes across the Poisson–Gaussian processes. We expand our definition of the short rate process as follows:

$$\Delta r_t = k(\theta_t - r_t)\,\Delta t + v_t\,\Delta z + J_t\,\Delta\pi(h_t),$$

$$\text{States}[\theta_t, v_t, h_t, J_t] = \begin{cases} \theta_1, v_1, h_1, J_1] \\ \theta_2, v_2, h_2, J_2] \end{cases},$$

$$\text{Transition matrix} = \begin{pmatrix} \lambda_1 & 1-\lambda_1 \\ 1-\lambda_2 & \lambda_2 \end{pmatrix}.$$

In addition to the usual parameters defined in Eq. (2.1), we provide for two regimes for the mean rate of interest θ_t, and for Gaussian volatility v_t. The mean value θ_t takes two values, $\{\theta_1, \theta_2\}$, and volatility v_t also takes two values $\{v_1, v_2\}$. Likewise, the jump parameters, h_t and J_t also switch between the two states. Changes from one regime to the other are generated via the transition matrix where λ_1 denotes the probability of remaining in state 1;

likewise, λ_2 denotes the probability of remaining in state 2. As discussed earlier, market overreaction causes temporary deviations from the target through the Poisson process $\mathrm{d}\pi$. This combination of regime switching and jumps captures three features. First, regime changes introduce persistent changes in target rates. Second, short term deviations from targets are embedded in the jump process $\mathrm{d}\pi$. Thus, we capture both features detailed in Balduzzi et al. (1998a). Finally, regime switching in volatility provides for stochastic volatility with persistence, such as in the model of Naik and Lee (1993). The jump size J_t is assumed to be distributed normally as follows: $J_t \sim \mathrm{N}[0, \gamma_t^2]$.

In order to estimate this system, we extend the discrete-time estimation model to account for regime changes. The transition density in each state is

$$\begin{aligned} f[r(s)|r(t)] = q_t \exp\left(\frac{-(r(s) - r(t) - k(\theta_t - r(t))\,\Delta t)^2}{2(v_t^2\,\Delta t + \gamma_t^2)}\right) \\ \times \frac{1}{\sqrt{2\pi(v_t^2\,\Delta t + \gamma_t^2)}} + (1 - q_t) \\ \times \exp\left(\frac{-(r(s) - r(t) - k(\theta_t - r(t))\,\Delta t)^2}{2v_t^2\,\Delta t}\right) \frac{1}{\sqrt{2\pi v_t^2\,\Delta t}} \end{aligned} \tag{4.1}$$

which is the mixture density for a two-term version of the Poisson–Gaussian transition density, with $q_t = h_t\,\Delta t$. Here $s > t$, i.e. time s is the observation after time t. We implement Hamilton's method for estimation of the regime-switching model. To begin with, we write the Markov probabilities in logit form as follows:

$$\lambda_i = \frac{\exp(\alpha_i)}{1 + \exp(\alpha_i)}, \quad i = 1, 2. \tag{4.2}$$

The time-varying ex-ante probabilities for states 1 and 2, $[p_1(s), p_2(s)]$ are specified in terms of the ex-post probabilities from the previous period, denoted $[\hat{p}_1(t), \hat{p}_2(t)]$, and the Markov chain parameters (λ_1, λ_2):

$$p_1(s) = \lambda_1 \hat{p}_1(t) + (1 - \lambda_2)\hat{p}_2(t),$$

$$p_2(s) = 1 - p_1(s).$$

The mixture distribution is then given by

$$f[r(s)] = p_1(s) f[r(s)|r(t), i = 1] + p_2(s) f[r(s)|r(t),\ i = 2].$$

Table 11
Modeling the short rate with jumps and regime-switching[a]

Parameter	Estimate	T-statistic
k	0.4930	1.54
θ_1	0.1139	0.85
θ_2	0.0142	0.46
$\sigma_1\sqrt{\Delta}$	0.0047	15.90
$\sigma_2\sqrt{\Delta}$	0.0005	8.83
q_1	0.0885	1.89
q_2	0.6524	9.51
γ_1	0.0117	3.83
γ_2	0.0019	19.47
α_1	1.2411	7.42
α_2	1.5559	10.92
Log-likelihood	12725.22	

[a]We present results for the estimation of the Poisson–Gaussian model allowing for changes in regimes. The system switches between two regimes, $i=1,2$, and in each regime follows a jump-diffusion process, estimated via Eq. (4.1). In each regime, the parameters for the mean (θ_i) of the process, diffusion volatility (σ_i), jump frequency (q_i) and size (γ_i) are varied. The Markov chain is defined by the switching probabilites in Eq. (4.2), with parameters α_i. Estimation is undertaken using maximum-likelihood.

Updating of the ex-post probabilities follows Bayes' Theorem:

$$\hat{p}_1(s) = \frac{p_1(s) f[r(s)|r(t), \quad i=1]}{f[r(s)]},$$

$$\hat{p}_2(s) = \frac{p_2(s) f[r(s)|r(t), \quad i=2]}{f[r(s)]}.$$

Parameter estimation obtains from a maximization of the log-likelihood function

$$\{k, \theta_1, \theta_2, \sigma_1, \sigma_2, q_1, q_2, \gamma_1, \gamma_2, \alpha_1, \alpha_2\}^* = \operatorname{argmax} \sum_{t=1}^{T} \log[f(r_t)].$$

Maximum likelihood estimation results are provided in Table 11.

The results from the model show that there are two regimes with high and low interest rate levels. The behavior of interest rate volatility is quite different in the two regimes. When interest rates are high (regime 1), volatility is higher, reflecting the level-dependence of interest rate volatility. Deviations from targets are infrequent but large, on the order of more than 100 basis points. Clearly at high rates, a substantial portion of the changes in interest rates comes from discontinuous moves in rates. When interest rates are low, volatility is also low, and while there are frequent jumps from target deviations, these are of small magnitude. Overall, in the low short rate regime, interest rates evidence much smoother behavior. For the sample period considered, simple calculations show that a majority of the time is spent in the low rate regime. We conclude that conditioning on the interest rate regime is important in determining the nature of the jump model used.

5. Concluding comments

This paper explores surprise elements in the fixed-income markets using Fed Funds data. We conclude that jumps are an essential component of interest rate models. Several examples of questions that may be explored using jump processes are provided in this paper.

The evidence in favor of jump models of the Fed Funds rate appears overwhelming. First, enhancement of the diffusion model with jumps resulted in a significant improvement in statistical fit, supporting mixed models with ARCH and jump effects. Second, the jump model lends itself easily to extended analysis of the impact of information variables, such as the meetings of the Fed Open Market Committee. We found evidence that the two-day meetings of the Fed resulted in information surprises to the market. Third, we were able to use the jump model to examine day-of-week effects, and found these to be quite significant. Wednesdays and Thursdays evidence a much higher likelihood of jumps than other days of the week, since option expiry effects may result in sharp market movements. Fourth, recent research has found that the drift term in the stochastic process for interest rates appears to be non-linear. The addition of a jump process diminishes the extent of non-linearity. Fifth, a study of the term-structure properties of the moments suggests the need for jump processes. Sixth, our model detected overreaction in the bond markets, and suggests a failure of the martingale hypothesis for Fed funds rates. Finally, the jump model lends itself to a simple extension in which regime switching is modeled; the empirical results suggest that the short rate evidences dichotomous behavior, discontinuous in one regime, and smooth in the other.

It is worthwhile suggesting further avenues of research, which would benefit from the framework of this paper. First, an extensive examination of which type of information surprise causes jumps is an open question. Locating jumps in the data and associating them with market events is one way of addressing this question (see Balduzzi et al. (1998c), Green (1998) and Piazzesi (1999) for recent work in this direction). Second, a question of importance is whether Fed actions are endogenous or exogenous to the interest rate markets. This aspect is a strong determinant in the choice of the modeling framework (see Balduzzi et al., 1997). Third, rather than model jumps in the level of the interest rate, modeling jumps in the mean and volatility of the short rate is an alternate approach (see Naik and Lee, 1993). Fourth, Heston 1995 employs a gamma process as an alternative to the Poisson–Gaussian framework. A comparison of this model with the one in this paper would be insightful. Fifth, this work may be related to the work of Brandt and Santa-Clara (1998), who develop a method of estimation using simulated transition density functions. Their work relates to diffusions only, and hence may be extended to jump-diffusions and then confirmed using the closed-form results of this paper. Finally, examining very short frequency intra-day data may reveal better the possible causes of jumps in bond yields. We leave this rich menu of research projects for future work.

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Appendix: Deriving moments from the characteristic function

To obtain the moments, differentiate the characteristic function successively with respect to s. Let μ_n denote the nth moment, and F_n be the nth derivative of F with respect to s, i.e. $F_n = \partial F/\partial s$. Then $\mu_n = 1/i^n F_n(s=0)$. Likewise let A_n, B_n be the nth derivatives of A and B, respectively, with respect to s. First let us compute the A_n. Substituting for B in A, we can write A as

$$A(T;s) = \int \left(k\theta \mathrm{i}s\mathrm{e}^{-kT} - \frac{1}{2}v^2 s^2 \mathrm{e}^{-2kT} + h\mathrm{E}\left[\mathrm{e}^{J\mathrm{i}s\mathrm{e}^{-kT}} - 1\right]\right) \mathrm{d}T.$$

Then,

$$\frac{\mathrm{d}A}{\mathrm{d}s} = \int [k\theta \mathrm{i}\mathrm{e}^{-kT} - v^2 s\mathrm{e}^{-2kT} + h\mathrm{i}\mathrm{e}^{-kT}\mathrm{E}[J\mathrm{e}^{J\mathrm{i}s\mathrm{e}^{-kT}}]]\,\mathrm{d}T,$$

$$\frac{d^2A}{ds^2} = \int [-v^2 e^{-2kT} - h e^{-2kT} E[J^2 e^{Jise^{-kT}}]] \, dT,$$

$$\frac{d^3A}{ds^3} = \int [-ih e^{-3kT} E[J^3 e^{Jise^{-kT}}]] \, dT,$$

$$\frac{d^4A}{ds^4} = \int [-ih e^{-4kT} E[J^4 e^{Jise^{-kT}}]] \, dT$$

which makes use of the fact that the integral is bounded and the expectation E(.) is also bounded. We can also compute the derivatives of A evaluated at $s=0$, which are:

$$\left(\frac{dA}{ds}\right)_{s=0} = \int i[k\theta e^{-kT} + h e^{-kT} E[J]] \, dT$$

$$= i\left(-\theta e^{-kT} - \frac{h}{k} E[J] e^{-kT}\right) + c_1.$$

Using the fact that $A(T=0;s)=0$, we get that $c_1 = \theta + (h/k)E[J]$, which when substituted back gives us:

$$\left(\frac{dA}{ds}\right)_{s=0} = i\left(\left(\theta + \frac{h}{k} E[J]\right)(1 - e^{-kT})\right).$$

In like fashion, we can obtain the other derivatives evaluated at $s=0$:

$$\left(\frac{d^2A}{ds^2}\right)_{s=0} = -\frac{v^2 + hE[J^2]}{2k}(1 - e^{-2kT}),$$

$$\left(\frac{d^3A}{ds^3}\right)_{s=0} = -ihE[J^3]\left(\frac{1 - e^{-3kT}}{3k}\right),$$

$$\left(\frac{d^4A}{ds^4}\right)_{s=0} = hE[J^4]\left(\frac{1 - e^{-4kT}}{4k}\right)$$

and the derivatives of B with respect to s:

$$\frac{dB}{ds} = ie^{-kT}, \quad \frac{d^2B}{ds^2} = \frac{d^3B}{ds^3} = \frac{d^4B}{ds^4} = 0.$$

We can write the intermediate value

$$\left(\frac{\mathrm{d}A}{\mathrm{d}s}+r\frac{\mathrm{d}B}{\mathrm{d}s}\right)_{s=0}=\mathrm{i}\left(\left(\theta+\frac{h\mathrm{E}[J]}{k}\right)\left(1-\mathrm{e}^{-kT}\right)+r\mathrm{e}^{-kT}\right)=\mathrm{i}\mu_1.$$

We can now evaluate the moments for the distribution of the interest rate r which are given by $\mu_n = 1/i^n F_n(s=0)$. Using these results, allows lengthy and tedious algebraic manipulation which gives us the moments presented in Section 2.3.

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