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The Design and Risk Management of Structured Finance Vehicles

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Abstract: Special investment vehicles (SIVs), extremely popular financial structures for the creation of highly-rated tranched securities, experienced spectacular demise in the 2007–2008 financial crisis. These financial vehicles epitomize the shadow banking sector, characterized by high leverage, undiversified asset pools, and long-dated assets supported by short-term debt, thus bearing material rollover risk on their liabilities which led to defeasance. This paper models these vehicles, and shows that imposing leverage risk control triggers can be optimal for all capital providers, though they may not always be appropriate. The efficacy of these risk controls varies depending on anticipated asset volatility and fire-sale discounts on defeasance. Despite risk management controls, we show that a high failure rate is inherent in the design of these vehicles, and may be mitigated to some extent by including contingent capital provisions in the ex-ante covenants. Post the recent subprime financial crisis, we inform the creation of safer SIVs in structured finance, and propose avenues of mitigating risks faced by senior debt through deleveraging policies in the form of leverage risk controls and contingent capital.

Keywords: special investment vehicle; structured finance; leverage risk controls; contingent capital

1. Introduction

In the mid-2000s, market segmentation and an ever-increasing demand for safe debt resulted in a marked shift in the supply of investable debt from the commercial banking system to the shadow banking system [1], which provided presumably safe assets via special investment vehicles (SIVs).

This mode of financing peaked in 2007, at approximately $22 trillion [2].[1] Much of this debt was rated AAA, and deemed safe. There were twenty-nine major structured investment vehicles (SIVs) created before the financial crisis, none of which remain today, that held an estimated $400 billion in assets [3]. Senior debt issued by these SIVs, which were a special type of special purpose vehicle (SPV), sustained an average 50% loss, belying their supposed safety, and lower priority capital notes experienced almost 100% loss [4]. Our purpose is to discuss the design and risks inherent in structured finance vehicles, to point out their weaknesses, and to provide recommendations for improvement in the design and risk management of these highly leveraged vehicles, keeping expected losses at acceptable levels for all participants in the capital structure of the SIV.

Structured-finance balance sheets comprise a pool of assets (most often mortgage-backed securities and other asset-backed securities, with rating restrictions). The liabilities that fund these assets are tranched, with senior and mezzanine notes being supported by subordinated capital notes, also known as the equity tranche. The bulk of the financing comes from senior debt,

[1] For reference, traditional banking liabilities at this time were approximately $14 trillion [2].

usually comprising 85%–95% of the financing (e.g., see [5] (p. 29)), and with the view to be rated as AAA by the major accredited rating agencies based on stressed simulation models that target expected percentage losses not exceeding 0.01%.[2] This pooling and tranching process is fraught with asymmetric information and lemons issues, as well as with liquidity problems. When the markets for collateral assets experience shocks, widening spreads on mortgages and asset-backed securities can lead to the demise of these structured finance vehicles due to their high leverage and asset-liability gap (i.e., the funding of longer-duration assets with shorter-duration debt).

Every limited liability corporation manages leverage, attempting to reach some optimal level, with banks and other financial institutions taking on more leverage than do manufacturing or tech firms. The SIVs we consider here are less diversified in their asset pools and are sometimes more highly leveraged than commercial banks, which are regulated to a leverage ratio of assets to equity of 11–12 times (based on risk-based capital of 8%–9% [8]). In contrast, a few of the SIVs in the crisis had leverage ratios that exceeded this range.[3] SIVs also face rollover risk in their entire liability pool, which is usually not the case for banks, which are insured by the Federal Deposit Insurance Corporation (FDIC). Hence, in order to manage this risk, SIVs will impose a leverage risk constraint, stipulating that the vehicle be shuttered in an orderly manner if leverage rises above a given level. This risk control is designed to limit losses to debt holders, but might also lead to a greater incidence of demise amongst these vehicles. Indeed, in the financial crisis, we saw that a majority of these vehicles of various types did indeed enter defeasance. This introduces the problem of asset liquidation, which must often offer at a discount. We account for these issues in our modeling to determine the optimal risk management prescriptions for SIVs.

Overall, this paper explores SIV design and risk management, assuming these issues as given. This paper is different from others in the literature in that it focuses on potential risks borne from the risk controls themselves, and also recommends a better ex-ante structure and risk management covenants to manage these vehicles in times of dropping asset values. Because optimal structured finance design can be sensitive to even small changes in the underlying parameters, more appropriate dynamic risk controls or alternate capital structures are predicated, and we will discuss these in ensuing sections.

The main contributions of this paper are as follows. First, we show that expected percentage losses to senior-debt holders may be managed by the introduction of a maximum leverage threshold, where the assets-to-debt ratio must remain above a prescribed level (denoted K). Most SIVs created before the crisis tended to have one or more leverage constraints of this nature, also referred to as market-value triggers. If violated, the vehicle's assets are sold down in an orderly manner to repay outstanding debt. This is known as "defeasance" of the vehicle. We show that ex-ante expected losses can be mitigated when such a leverage threshold is introduced optimally, thereby justifying the imposition of these risk controls, provided they are properly set.

Second, designing a SIV involves a delicate tradeoff considering asset volatility as well as the anticipated fire-sale discounts, and becomes a particularly difficult exercise in the presence of high estimation error.[4] This consideration is particularly important given the highly uncertain nature of fire-sale discounts in times of distress, whereby one SIV's defeasance may easily trigger that of another SIV, pushing up fire-sale discounts even higher. For instance, Cheyne Finance recovered 44% of par value in initial liquidation rounds [10],[5] and Sigma Finance recovered 15% [3]. Overall, stress

[2] This cutoff is based on the *Rating Methodology* guide published by Moody's Investors Service. See Exhibit 2 of [6] (p.10). See also [7] for issues relating to rating SIVs.

[3] More generally, SIVs form a subset of special purpose vehicles (SPVs), of which some, such as special derivatives vehicles and closed-end yield funds, were leveraged in the range of 12–30 times. However, most of these vehicles also took leverage in a lower range, less than 15x.

[4] For more on estimating fire sale discounts, see [9].

[5] Cheyne was one of the largest SIVs created in 2005 before the onset of the financial crisis, and was the first to include subprime assets in its pool.

tests of SIV design must not only account for swings in spread volatility (i.e., extreme pool risk) but must also account for the potentially vast swings in fire-sale discounts under defeasance.[6] We show that the qualitative structural prescriptions for an SIV's risk controls are not affected by the levels of asset volatility or fire sale discounts, but of course, the exact quantitative specifications will vary.

Third, we consider contingent capital as one approach to enhancing the risk management of a SIV. We show that pre-committed equity infusions reduce ex-ante expected percentage losses to both senior- and capital-note holders, reduce the probability of defeasance, and extend the expected life of the vehicle. In addition, we believe that a SIV sponsor's ex-ante commitment to provide contingent capital sends a strong signal to the markets that the quality of their SIV and under collateral is better than that of vehicles that do not incorporate a mandated capital infusion. Contingent capital is analogous to deposit insurance which has the added benefit of mitigating rollover risk [14], particularly when applied prior to a state of financial crisis [15], which in the SIV setting, means prior to approaching defeasance.

Our paper may also be framed as an analysis of the risk management of economic catastrophe bonds. For instance, the senior tranches of securitized pools have been shown to be similar to economic catastrophe bonds, failing under extreme situations, but offering lower compensation than investors should require [16]. Through the use of a one-time committed capital infusion, we mitigate the senior notes' likeness to these economic catastrophe bonds. Furthermore, related work has shown that small errors in parameter estimates of the collateral assets can lead to a large variation in the risk assessment of the senior tranches [17]. We show that risk controls on the liability side are also very relevant to this assessment, i.e., small changes in the leverage threshold can have a material impact on expected losses.

The rest of the paper proceeds as follows. In Section 2, we present the model set up, and in Section 3 we describe the implementation of our model using trees. In Section 4 we undertake a numerical analysis of our base model and present results, including validation of the results for varying asset volatility and fire sale discounts. In Section 5, we demonstrate how to mitigate risk to senior notes and capital notes via contingent capital, an innovation that was not implemented ex-ante in SIVs, and that we prescribe moving forward. In Section 6, we discuss and conclude.

2. Model

2.1. Basic Set up

We develop a parsimonious model of structured finance and SIV design. At time t, the aggregate value of the asset pool is denoted as $A(t)$, the senior-note liabilities are denoted as $B(t)$, and the capital-note liabilities are denote as $C(t)$, with $A(t) = B(t) + C(t)$. For simplicity, we impose only two liability tranches, the senior notes (which we refer to as the debt of the SIV) and the capital notes (which we refer to as the equity), abstracting away from multiple layers of mezzanine tranches in between (which, in any case, did not exist in all SIVs). For simplicity, we assume that senior debt is issued as a pure-discount note, especially given that it is short-term, and the initial face values of the capital and senior notes are denoted D_C and D_B, respectively. At inception (i.e., at time $t = 0$), $B(0) < D_B$ represents the initial price of senior notes with face value D_B, and the residual value of the liabilities represents the upfront equity financing, i.e., $C(0) = D_C = A(0) - B(0)$.

Investors in senior notes seek safe assets with assured returns, and hence, SIVs are designed with the intent to attain a high quality credit rating for the senior notes. We will assume here that this reflects a pre-specified expected loss from default [18]. For our examples we assume an expected

[6] Fire sales also matter as they result in increasing correlations of distressed assets (see [11–13]), which further increases overall pool risk.

loss of $L_0 = 0.10\%$, i.e., 10 bps (of face value of senior debt). The expected loss to these investors is critically dependent on the following factors:

1. The credit quality of the asset pool, i.e., its volatility σ, defined later in the stochastic process Equation (3),
2. The size of the senior tranche (D_B) and subordinated capital notes tranche (D_C), and
3. The risk controls outlined by the SIV, i.e., the leverage control threshold K, which sets the minimum acceptable level of the ratio of assets to senior debt. If this risk control is violated, then the vehicle is usually terminated and an orderly sell down of the assets is triggered.

The capital-note holders retain the residual after repayment of the senior notes and management fees borne by the SIV.[7] Capital-note holders bear first losses on the SIV's assets. The SIV makes a spread between the rate of return on the assets and the rate of interest paid on the senior notes, from which the capital note holders are compensated. This spread arises because the assets held are of longer maturity than that of the senior notes, which are periodically rolled over. Hence, the maturity of the senior notes is shorter than that of the capital notes and the weighted average maturity of all assets and liabilities of the SIV.

2.2. Risk Controls and Defeasance

To afford the senior notes additional protection, the SIV covenants typically include safeguards pertaining to asset quality in addition to requirements regarding the duration and liquidity of the assets in the collateral pool. In addition to these asset restrictions, the SIV is monitored on an ongoing basis, primarily through a leverage test, whereby the ratio of collateral value to senior-note obligations must meet a pre-specified cutoff.[8] Failure to meet these requirements forces the SIV into "defeasance", whereby the assets of the SIV must be sold to wind down operations. The proceeds are used to first pay off the senior note holders, with the residual used to pay off the capital (equity) note holders.

We define a SIV's (inverse) leverage ratio covenant at any point in time t as

$$\frac{A(t)}{D_B} \geq K \tag{1}$$

where K is the lower bound on the ratio of collateral assets, A(t), to senior notes, D_B, that is permitted by the SIV. For example, a SIV with assets worth 100 and senior notes with a face value of 92, has a leverage ratio of $100/92 = 1.087$. Suppose a lower limit of $K = 1.04$ is placed on the leverage ratio. Then the SIV enters defeasance (i.e., enforcement mode) when $A(t) = D_B \cdot K = 92 \times 1.04 = 95.68$. We denote this boundary as $H = D_B \cdot K$. Assuming a frictionless market with no fire-sale discounts, at any time $t < T$, senior-note holders are due the riskless value $D_B e^{-r(T-t)}$, (here r is the risk free rate, and the payout is the value of the discount bond, which has accreted value since inception, until rollover maturity T). Capital-note holders are left with a payout that is far less than D_C; i.e., a 4.32% drop in the value of the assets held by the SIV results in a disproportionately greater loss to the capital-note investors, who suffer first losses.

In reality, defeasance likely occurs when markets are under stress and other financial institutions (including other SIVs) are also attempting to unwind. Thus, assets are unlikely to be sold at full market value but rather at fire-sale prices, at some percentage discount δ. Hence, the anticipated recovery is only a fraction $(1 - \delta) < 1$ of the assets' value at defeasance, i.e., $((1 - \delta) \cdot D_B \cdot K)$.

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7 If the capital-note holders are the sponsors/managers of the SIV, then they also collect management fees.

8 Other tests are also imposed, such as those on liquidity and asset-pool composition, but these tests are less likely to trigger defeasance than the leverage test, which is essentially a trigger corresponding to a pre-specified drop in collateral value. Thus, the SIV risk-model trigger is akin to a barrier in the [19] class of structural default models.

2.3. Rollover Risk

An SIV invests in an asset pool of securities that have much higher duration than that of its liabilities. Senior notes mature and are rolled over at intervals ranging from as small as one month to a year (and sometimes more). Rollover may not occur for two reasons. For one, if assets fall to a level ($A(t) \leq D_B K$) that violates the leverage ratio covenant, the vehicle will enter defeasance prior to any rollover decision, as discussed in the previous section.

Second, conditional on the leverage ratio not being violated, i.e., $A(t) > D_B K, \forall t < T$, senior-debt investors will decline to roll over their investment if, on the roll date, the level of the SIV's assets are insufficient to ensure that the ex-ante expected losses to senior notes are no greater than L_0; i.e., if $A(T) \leq A^*$, where A^* is the minimum level of assets required to keep expected losses on senior notes D_B within required quality levels (i.e., $E(L_B) \leq L_0$). In short, the vehicle fails to survive when senior debt holders refuse to roll over their debt, because the expected percentage losses have now exceeded L_0.

We define $D_B^* > D_B$ as the level of maximal debt, given K, satisfying the constraint $E(L_B) \leq L_0$. That is, at D_B^*, the loss constraint is exactly met, $E(L_B) = L_0$. Note that D_B^* is a function of K, i.e., may be written as $D_B^*[K]$. This implies a relationship between A^* and D_B^*, given K, which is that

$$A^*[K] = A_0 \cdot \frac{D_B}{D_B^*[K]} \leq A_0 \tag{2}$$

To summarize, rollover occurs when assets at maturity are greater than A^*. Thus, if assets do not decline in value by more than $A_0 - A^*$, then rollover occurs, and we may assume that the equity holders maintain their equity to support assets of A_0 so that the problem then repeats itself, when debt is rolled over. Capital note holders will withdraw excess equity so as to maximize return on equity. Because keeping excess equity reduces leverage and reduces the expected return on equity, withdrawal of excess equity capital reinitializes the SIV to its original risk-return trade-off. This configuration leads to a series of one period repeated problems, and the value of debt at inception of the rollover period is based on the outcomes of a single period. This approach makes our solution simple, as we need only to solve a one-period problem to understand the ongoing risk and return behavior of these SIVs.

While we accommodate for rollover risk in the model, we do not include the more complicated case of staggered rollover dates for the senior debt in the SIV, which occurs when where there are multiple time tranches with varying rollover dates. This paper was aimed at focusing on the design of risk management controls for SIVs. We take up the case of staggered rollover dates in a follow up paper that focuses mostly on that particular risk, and we offer a novel solution to that form of rollover risk (see [20]). For earlier work in the topic of rollover risk, not specific to SIVs, see [21,22].

2.4. The Stochastic Asset Process

We begin by modeling the asset-price $A(t)$, defined by the following process:

$$\frac{dA(t)}{A(t)} = \mu\, dt + \sigma\, dW(t) \tag{3}$$

where $\mu > r > 0$ is the expected growth rate of the assets over time, used to pay returns to both classes of note holders. The risk free rate of interest is denoted by r. Asset volatility is modulated by coefficient σ, and $W(t)$ is a standard Wiener process. Under this process the evolution of the asset price may be written in non-differential form as:

$$A(t+h) = A(t) \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) h + \sigma W(h)\right] \tag{4}$$

where h is a discrete time step for the model. Therefore, the terminal value of $A(T)$ follows the lognormal distribution. Valuation of the capital and senior notes is achieved under the risk-neutral framework by replacing μ with risk free rate r in the expressions above.

3. Model Implementation

To examine the role of risk management covenant K, we price senior notes and capital notes with and without the presence of this covenant (leverage ratio trigger). We denote the defeasance trigger level as $H = D_B \cdot K$, and valuing the liabilities (senior notes and capital notes) of the SIV with the leverage trigger is analogous to a barrier option pricing problem. In addition to valuation of the securities in the structure, we are also interested in the risk features of the notes issued, such as expected percentage loss on the securities, as well as the probability of defeasance at the barrier and the probability with which senior debt holders decline to roll over at maturity. Furthermore, we may also be interested in the moments of the returns to these securities. Some of these quantities are produced under the risk-neutral probability measure and others are computed under the physical probability measure.

For ease of computation and tractability, we implement the model using binomial trees. We describe our approach briefly so that the reader may have sufficient detail with which to replicate the model.

The tree is set up with constant time step h (time is denoted in years). An asset price $A(t)$ at any node will evolve to two possible values, $A(t+h) = \{u \cdot A(t), d \cdot A(t)\}$, with $u = e^{\sigma\sqrt{h}}$ and $d = e^{-\sigma\sqrt{h}} = 1/u$.[9] The risk-neutral probability of an up move is $q = \frac{R-d}{u-d}$, where $R = e^{rh}$. With this set up, as shown in [23], the evolution of the stochastic process for $A(t)$ is arbitrage-free. The tree, then, becomes the foundation for the valuation of any security dependent on the value of the assets, $A(t)$.

3.1. Valuing Senior Debt (P_B)

Senior debt is valued on the tree using the standard backward recursion approach, where we build the tree to rollover maturity T. At the maturity nodes we obtain the terminal payoffs to senior debt, as follows:

$$B(T) = \begin{cases} D_B & \text{if } A(T) \geq A^* \\ \min[D_B, A(T)(1-\delta)] & \text{if } H \leq A(T) < A^* \\ \min[D_B, H(1-\delta)] & \text{if } A(T) < H \end{cases} \tag{5}$$

Backward recursion is then applied to find the values of senior debt at all times $0 \leq t < T$, through the following pricing recursion:

$$B(t) = \frac{1}{R}[q \cdot B_u(t+h) + (1-q) \cdot B_d(t+h)] \tag{6}$$

where the subscripts u, d denote the up and down nodes emanating from $B(t)$. At each stage, we also check where defeasance has occurred, i.e., whether $A(t) \leq H$. In such cases, we set

$$B(t) = \min[B(t), H(1-\delta)] \tag{7}$$

and the initial price of senior debt is denoted $P_B \equiv B(0)$, which is determined after backward recursion on the tree is undertaken using the equations above, from time $t = T$ to time $t = 0$.

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9 e is understood to represent the base of the natural logarithm, and is approximately equal to 2.71828.

3.2. Expected Percentage Loss on Senior Debt ($E(L_B)$)

In similar fashion, we are also able to compute the expected losses on senior notes, the equations for which are as follows:

$$L_B(T) = \begin{cases} 0 & \text{if } A(T) \geq A^* \\ \max[0, D_B - A(T)(1-\delta)] & \text{if } H < A(T) < A^* \\ \max[0, D_B - H(1-\delta)] & \text{if } A(T) \leq H \end{cases} \tag{8}$$

Backward recursion gives the expected loss value as follows:

$$L_B(t) = \frac{1}{R}[q \cdot L_{Bu}(t+h) + (1-q) \cdot L_{Bd}(t+h)] \tag{9}$$

and if $A(t) \leq H$, then

$$L_B(t) = \max[0, D_B/R^{T-t} - H(1-\delta)] \tag{10}$$

At rollover maturity, we compute losses based on the face value of debt D_B, as this is the amount owed to the senior note holders. That is, even though the initial price paid for the debt is $B(0) < D_B$, D_B reflects the loss of accrued interest. For defeasance at times $t < T$, we compute losses based on the discounted value of debt, once again, accounting for accrued interest in the loss computation, as may be seen in Equation (10). The expected percentage loss based on face value is

$$E(L_B) = \frac{L_B(0)}{D_B} \tag{11}$$

and the goal is to design our vehicle such that $E(L_B) \leq L_0$.

3.3. Valuing Equity (D_C) and Expected Percentage Loss to Equity ($E(L_C)$)

The initial value of equity is required to be $D_C = A(0) - B(0)$ and the losses to equity may be assessed at maturity as follows:

$$L_C(T) = \begin{cases} 0 & \text{if } A(T) \geq A^* \\ D_C - \max[0, A(T)(1-\delta) - D_B] & \text{if } H < A(T) < A^* \\ D_C - \max[0, H(1-\delta) - D_B] & \text{if } A(T) \leq H \end{cases} \tag{12}$$

Recursion gives the expected loss value as follows:

$$L_C(t) = \frac{1}{R}[q \cdot L_{Cu}(t+h) + (1-q) \cdot L_{Cd}(t+h)] \tag{13}$$

and if $A(t) \leq H$, then

$$L_C(t) = D_C - \max[0, H(1-\delta) - D_B/R^{T-t}] \tag{14}$$

The expected percentage loss is

$$E(L_C) = \frac{L_C(0)}{D_C} \tag{15}$$

and we expect that $E(L_C) >> E(L_B)$ because capital notes are analogous to equity and senior notes are akin to debt in a standard capital structure. Here $L_C(0)$ is in fact the value of a put option on capital notes, where the put pays off whenever these investors do not recoup their investment (this is not the expected loss conditional on a loss occurring).

3.4. Calculating Probability of Defeasance at Barrier and Maturity

The risk-neutral pricing tree we construct for valuing the notes may also be used to determine the risk-neutral probability of defeasance at the barrier (denoted PD_K) and at maturity (denoted PD_T).[10] Unlike the valuation of notes in the preceding sections, which was undertaken via backward recursion, these probabilities are computed by making a forward pass through the n-period tree, from period zero through period n, where of course, $T = nh$.

Recalling that the probability of an upshift on the tree is q and a downshift is $1 - q$, we denote the probability of reaching node i in period j as p_{ij}. Since the time step per period is h, the time elapsed by period j is equal to $j \cdot h$. We also note that at the end of period j, there are $j + 1$ nodes (i.e., economic states), and we enumerate the nodes in period j from $i = 0, 1, 2, ..., j$.

At time $t = 0$, the single node has a probability of $p_{00} = 1$, because this state must occur with certainty. Emanating from this node are two nodes at time $t = h$ (period $j = 1$); the upper node occurs with probability $p_{01} = q$ and the lower node with probability $p_{11} = 1 - q$. From here on we generate probabilities using forward accumulation, as follows:

$$\begin{aligned} p_{0j} &= p_{0,j-1} \cdot q, \quad i = 0 \\ p_{ij} &= p_{i-1,j-1} \cdot (1-q) + p_{i,j-1} \cdot q, \quad 0 < i < j \\ p_{jj} &= p_{j-1,j-1} \cdot (1-q), \quad i = j \end{aligned} \tag{16}$$

During the forward pass, we also compute the probability of defeasance in period j, which occurs when $A_{ij} \leq H$ for any node (i, j). The first passage time probability to the defeasance barrier is the probability that defeasance occurs in period j, given that defeasance has not occurred at any period prior to period j. During the forward pass, at period j, we compute the first passage time probability as follows:

$$f_j = \sum_{\forall i: A_{ij} \leq H} p_{ij} \tag{17}$$

Once this is calculated and stored, we then set

$$p_{ij} = 0, \forall i : A_{ij} \leq H, \tag{18}$$

so that the ensuing periods, first-passage probabilities will be correctly computed using the forward pass Equation (16), accounting for the event of defeasance.

In short, the risk-neutral probabilities are computed by traversing the tree forward iterating over Equations (16)–(18), in sequence. The total probability of defeasance at the barrier is then given by the sum of first-passage probabilities over all periods j:

$$PD_K = \sum_{j=1}^{n} f_j \tag{19}$$

and the total probability of defeasance at maturity is

$$PD_T = \sum_{\forall i: A_{i,n} < A^*} p_{i,n} \tag{20}$$

The total risk-neutral probability of defeasance is

$$PD = PD_K + PD_T \tag{21}$$

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[10] We also report the physical probabilities of default (PDs), and we use these physical probabilities to calculate expected returns to senior- and capital-note investors.

We compute and report all of these values in our assessment of SIV design.

3.5. Expected Time to Defeasance

Using the probabilities p_{ij} the expected time to defeasance is the time to defeasance at each t weighted by the first passage probabilities, conditional on defeasance occurring. It can be computed as follows:

$$ETD = \frac{\sum_j (jh) f_j + T \cdot PD_T}{PD} \tag{22}$$

Of course, ETD shortens as the leverage barrier K increases.

4. Analysis

SIVs of the type considered here are highly-leveraged vehicles in that A_0/D_B tends to be close to 1. Correspondingly, A_0/D_C is a large multiple ranging from 10–30 times leverage. Since many such vehicles failed in the crisis, a natural question arises as to whether they were designed, ex-ante, to fail. We may also assess the properties of these SIVs, such as the expected losses for each tranche, the probability of defeasance, etc. We are also interested in asking whether risk controls on leverage (denoted by parameter K) would have mitigated the risks of these vehicles. We begin with simple examples to quantify the properties of SIVs.

4.1. Example: SIV without Risk Controls

At the outset of the SIV, the level of maximal debt D_B^* is chosen such that $E(L_B)/D_B = L_0$. This is the debt level that is achieved assuming that no leverage risk controls are imposed. We solve for D_B^* numerically, as the computational search is rapid and efficient.[11]

We consider the initial case in Table 1 as an example. Using these parameter values we compute the senior debt level D_B^* that makes $E(L_B) = L_0$. We then set actual face value of debt, D_B, to be lower than this amount, at 99% of D_B^*. Table 2 shows the outcomes for the resultant SIV.

Maximal permissible senior debt is $D_B^* = 87.9253$ on initial assets of $A_0 = 100$, and this discount debt has a price of $P_B \equiv B(0) = 86.0963$, implying an initial leverage of $A_0/D_C = 7.19$ (times capital (equity) notes). If we reduce debt to 99% of maximal allowed, the expected percentage loss on senior debt drops by half, to $E(L_B) = 0.0005$, and the expected percentage loss to capital note holders falls from 22% to 15%. The probability of defeasance (which can only occur at maturity since there is no defeasance trigger K) also declines at the lower level of debt, from 24% at maximal debt levels to 17% at just below maximal. Since there is no leverage barrier, defeasance cannot occur prior to maturity, hence the probability of defeasance at the barrier is $PD_K = 0$. Overall, the risks inherent in an SIV, particularly the risks borne by the capital note holders, are highly sensitive to small changes in parameter values.

Table 1. Initial case parameters.

Parameter	Value
Asset value (A_0)	100
Rollover maturity (T)	1 year
Expected return on assets (μ)	0.04
Volatility (σ)	0.03
Risk free rate (r)	0.02
Fire sale discount (δ)	0.10
Stipulated expected percentage loss (L_0)	0.0010
Number of periods on the pricing tree (n)	260

[11] All code in this paper is implemented using the `R` programming language.

Table 2. Results for the special investment vehicles (SIV) when no leverage controls are imposed, i.e., $K = 0$. Results are shown for the case when senior debt is at its maximal amount, D_B^*, with expected percentage losses on senior debt ($E(L_B)$) equal to $L_0 = 0.0010$ (10 bps). Results are also shown for the case where senior debt is set to 99% of the maximal amount.

Result	$D_B = 0.99D_B^*$	$D_B = D_B^*$
Leverage threshold (K)	0	0
Maximal debt face value (D_B^*)	87.9253	87.9253
Senior debt face value (D_B)	87.0460	87.9253
Price of senior notes, if risk free	85.3224	86.1843
Price of senior notes ($B(0)$) with defeasance risk	85.2831	86.0963
Price of capital notes (D_C)	14.7169	13.9037
Expected percentage loss to senior notes ($E(L_B)$)	0.0005	0.0010
Expected percentage loss to capital notes ($E(L_C)$)	0.1539	0.2206
Expected return to senior notes under physical probability (μ_B)	0.0301	0.0307
Expected return to capital notes under physical probability (μ_C)	0.1880	0.1608
Probability (risk-neutral) of defeasance at barrier (PD_K)	0	0
Probability (risk-neutral) of defeasance at maturity (PD_T)	0.1679	0.2374
Total risk-neutral probability of defeasance (PD)	0.1679	0.2374
Expected time to defeasance	1	1
Probability (physical) of defeasance at barrier (PD_K^{RN})	0	0
Probability (physical) of defeasance at maturity (PD_T^{RN})	0.0721	0.1173

Expected percentage losses to capital-note holders are around 15%–22% of their initial investment. However, the risk-neutral probability of no defeasance is more than 75%, and in these cases, the capital-note holders stand to gain from the upside of an at-the-money call option on the assets. Under the physical probability measure, assuming a growth rate of assets of $\mu = 4\%$ the expected return is around 16-18% compared to the risk free rate of $r = 2\%$.) The expected return to senior notes is around 3%.

4.2. Example: SIV with Risk Controls

As discussed earlier, the SIV may impose a leverage threshold level $K < \frac{A(0)}{D_B}$ to which asset values drop to trigger defeasance (i.e., orderly wind down) of the vehicle, requiring a sale of the assets, which leads to incurring an expected fire sale discount of $\delta = 0.10$ (recall base case parameters presented in Table 1). For illustration, we now consider a leverage trigger at $K = 1.05$, with results shown in Table 3.

Table 3. Results for the SIV when leverage controls *are* imposed, i.e., $K = 1.05$. Results are shown for the case when senior debt is at its maximal amount, D_B^*, with expected percentage losses on senior debt ($E(L_B)$) equal to $L_0 = 0.0010$ (10 bps). Results are also shown for the case where senior debt is set to 99% of the maximal amount.

Result	$D_B = 0.99D_B^*$	$D_B = D_B^*$
Leverage threshold (K)	1.05	1.05
Maximal debt face value (D_B^*)	87.9253	87.9253
Senior debt face value (D_B)	87.0460	87.9253
Price of senior notes, if risk free	85.3224	86.1843
Price of senior notes ($B(0)$) with defeasance risk	85.2831	86.0963
Price of capital notes (D_C)	14.7169	13.9037
Expected percentage loss to senior notes ($E(L_B)$)	0.0005	0.0010
Expected percentage loss to capital notes ($E(L_C)$)	0.1539	0.2206
Expected return to senior notes under physical probability (μ_B)	0.0301	0.0307
Expected return to capital notes under physical probability (μ_C)	0.1877	0.1598
Probability (risk-neutral) of defeasance at barrier (PD_K)	0.0003	0.0011
Probability (risk-neutral) of defeasance at maturity (PD_T)	0.1676	0.2363
Total probability of defeasance (PD)	0.1679	0.2374
Expected time to defeasance (ETD)	0.9998	0.9992
Probability (physical) of defeasance at barrier (PD_K^{RN})	0.0003	0.0014
Probability (physical) of defeasance at maturity (PD_T^{RN})	0.0721	0.1172

Since there is a leverage barrier, defeasance before maturity is possible, and we see that $PD_K > 0$ in both cases, and the expected time to defeasance $ETD < 1$. The resultant probability of defeasance at maturity PD_T is smaller when $K = 1.05$ versus when $K = 0$, but the total probability of defeasance PD remains the same. In the following sections we explore more aggressive leverage controls to assess the behavior of leveraged SIVs.

4.3. Role of Risk Controls

As we change leverage risk control $K > 1$, we obtain different SIVs, each with different risk-return properties. In order to make a fair comparison across these SIVs, we adjust the face value of senior debt so that the initial price of debt, $B(0) = P_B$, is the same across all compared vehicles. Therefore, if assets are $A_0 = 100$, and the price of senior debt is fixed at P_B, then across all structures, D_C (the size of the equity tranche) as well as initial leverage are the same. A natural question now arises as to what level of $K > 1$ makes senior debt holders and capital (equity) note holders better off than when there are no risk controls ($K = 0$). Is it possible for some $K > 1$, to achieve a Pareto-improving solution by mitigating deadweight losses incurred by either party?

To explore this question, we begin by fixing the initial price of debt P_B based on a face value of debt D_B that is 99% of the maximal allowed debt D_B^*, when $K = 0$. We then examine how the risk-return tradeoff to investors varies when anticipated fire sale discounts, δ, are low versus high. To set ideas, we begin with the base case when $\delta = 0.05$, depicted in Table 4.

Table 4. SIV outcomes when risk control K is varied. The fire sale discount is fixed at $\delta = 0.05$ and D_B is varied such that the price of debt is fixed across all cases to that when $K = 0$, and $D_B = 0.99 D_B^*$.

$\sigma = 0.03; \delta = 0.05$							
K	**0**	**1.005**	**1.015**	**1.025**	**1.035**	**1.04**	**1.045**
D_B	91.8819	91.8819	91.8815	91.8801	91.8639	91.865	91.8612
D_B^*	92.8100	92.8146	92.8277	92.8901	96.6184	96.1538	95.6938
P_B	90.0210	90.0210	90.0210	90.0210	90.0210	90.0210	90.0210
$E(L_B)$	0.0005	0.0005	0.0004	0.0004	0.0002	0.0002	0.0001
D_C	9.9790	9.9790	9.9790	9.9790	9.9790	9.9790	9.9790
$E(L_C)$	0.1480	0.1480	0.1480	0.1482	0.0227	0.0376	0.0604
PD_K	0.0000	0.0011	0.0034	0.0091	0.0230	0.0382	0.0623
PD_T	0.1679	0.1668	0.1645	0.1588	0.0000	0.0000	0.0000
PD	0.1679	0.1679	0.1679	0.1679	0.0230	0.0382	0.0623
ETD	1.0000	0.9988	0.9958	0.9864	0.7002	0.6614	0.6204
μ_B	0.0301	0.0301	0.0301	0.0300	0.0298	0.0297	0.0297
μ_C	0.2984	0.2984	0.2984	0.2986	0.3328	0.3293	0.3227
PD_K^{RN}	0.0000	0.0000	0.0000	0.0019	0.0062	0.0121	0.0230
PD_T^{RN}	0.0721	0.072	0.0716	0.0702	0.0000	0.0000	0.0000

We note that, as required, the expected percentage loss is less than L_0, 10 bps, and in fact in all cases, it is strictly within limits. We also note that the price of senior debt P_B is equal across all cases, i.e., levels of K, as is the value of capital notes D_C. Moreover, $PD_T = 0$ when $K \geq 1.035$, because at these levels, $H = A^*$, so if defeasance is to occur it can only happen prior to maturity.

The more important question, of course, is whether introduction of risk controls ($K > 1$) improves the SIV for senior and capital notes. First, as K increases, D_B decreases, keeping P_B fixed. Therefore, the continuous discount yield on senior debt ($\ln(D_B/P_B)$) declines, suggesting the credit quality improves. Second, the expected percentage loss on senior notes declines monotonically, but for capital note holders, the expected percentage loss declines initially, but begins to rise again as K continues to increase. Third, the mean return on senior notes remains the same; in contrast, the mean return on capital notes initially falls but then begins to rise when K increases to 1.035, which appears to be a sweet spot for the leverage control K. Finally, the probability of defeasance drops dramatically by $K = 1.035$, suggesting that risk controls do indeed help survival of the SIV. Overall, the results suggest that risk controls can not only enhance the quality of the securities issued by the

SIV, but can also begin to increase the risk to certain investors when the controls are too stringent. That is, the most effective point for the leverage threshold K is not too lax (i.e., not too close to 1), but not too strict either.

The intuition for this result is fairly natural, because the leverage risk barrier trades off the probability of default against the fire sale losses on default. As we increase K, the PD prior to maturity increases, but defeasance at higher asset values leads to lower fire sale loss.

4.4. Varying Fire Sale Discounts

We now explore the resulting SIVs and risk-return tradeoffs under varying expected fire-sale discounts. For comparison, we present Table 5 where we consider $\delta = \{0.02, 0.10\}$, values that span the earlier case of $\delta = 0.05$.

Table 5. SIV outcomes when risk control K is varied. The fire sale discount takes two values $\delta = \{0.02, 0.10\}$ and the price of debt is fixed across all cases to the price of debt when $K = 0$, and $D_B = 0.99D_B^*$.

			$\sigma = 0.03$; $\delta = 0.02$				
K	**0**	**1.002**	**1.006**	**1.01**	**1.014**	**1.016**	**1.018**
D_B	94.7834	94.7674	94.7676	94.7649	94.7586	94.7536	94.748
D_B^*	95.7408	99.8004	99.4036	99.0099	98.6193	98.4252	98.2318
P_B	92.8638	92.8638	92.8638	92.8638	92.8638	92.8638	92.8638
$E(L_B)$	0.0005	0.0003	0.0002	0.0002	0.0001	0.0000	0.0000
D_C	7.1362	7.1362	7.1362	7.1362	7.1362	7.1362	7.1362
$E(L_C)$	0.1407	0.0227	0.0320	0.0443	0.0595	0.0677	0.0771
PD_K	0.0000	0.0230	0.0325	0.0452	0.0623	0.0724	0.0847
PD_T	0.1679	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
PD	0.1679	0.0230	0.0325	0.0452	0.0623	0.0724	0.0847
ETD	1.0000	0.7002	0.6762	0.6497	0.6204	0.6030	0.5881
μ_B	0.0301	0.0298	0.0298	0.0297	0.0296	0.0296	0.0295
μ_C	0.4349	0.4554	0.4541	0.4523	0.4494	0.4474	0.4445
PD_K^{RN}	0.0000	0.0062	0.0097	0.0150	0.0230	0.0282	0.0348
PD_T^{RN}	0.0721	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
			$\sigma = 0.03$; $\delta = 0.10$				
K	**0**	**1.01**	**1.03**	**1.05**	**1.07**	**1.08**	**1.09**
D_B	87.046	87.046	87.046	87.046	87.0456	87.0447	87.0287
D_B^*	87.9253	87.9253	87.9254	87.9253	87.9322	87.9767	91.7431
P_B	85.2831	85.283	85.2831	85.283	85.2831	85.2831	85.2831
$E(L_B)$	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0003
D_C	14.7169	14.717	14.7169	14.717	14.7169	14.7169	14.7169
$E(L_C)$	0.1539	0.1539	0.1539	0.1539	0.1539	0.1540	0.0189
PD_K	0.0000	0.0000	0.0000	0.0003	0.0027	0.0076	0.0191
PD_T	0.1679	0.1679	0.1679	0.1676	0.1652	0.1604	0.0000
PD	0.1679	0.1679	0.1679	0.1679	0.1679	0.1679	0.0191
ETD	1.0000	1.0000	1.0000	0.9998	0.9967	0.9892	0.7093
μ_B	0.0301	0.0301	0.0301	0.0301	0.0301	0.0300	0.0298
μ_C	0.1880	0.1880	0.1880	0.1880	0.1881	0.1881	0.2341
PD_K^{RN}	0.0000	0.0000	0.0000	0.0000	0.0004	0.0015	0.0049
PD_T^{RN}	0.0721	0.0721	0.0721	0.0721	0.0717	0.0706	0.0000

We observe that under a lower discount of $\delta = 0.02$, a small increase in K above 1, e.g., at $K = 1.002$ seems to bring immediate benefit and also delivers the lowest probability of defeasance; though, compared to the case when $K = 0$ (i.e., no risk control), any level of risk control $K > 1$ appears to reduce the risks borne by both senior- and capital-note holders. The expected return to senior notes is again not affected materially by K, but the expected return to capital notes is best at lower levels of (keeping $K > 1$). In contrast, under a higher fire sale discount of $\delta = 0.10$, tighter risk controls are required to materially reduce the risk to investors in the SIV. Yield spreads decline and expected returns improve only at higher levels of K under this greater $\delta = 0.10$.

The intuition here is straightforward. Leverage risk controls force defeasance earlier than maturity, at a point when assets have not yet fallen drastically. When there is no defeasance barrier, asset values may continue to decline all the way to rollover maturity, precipitating huge losses. However, there is a tradeoff. The defeasance barrier prematurely cuts off the paths of asset prices that may recover eventually. It therefore raises the probability of defeasance, while limiting the losses when defeasance occurs. Since expected losses are a composite of the probability of defeasance and loss on defeasance, there is an optimal point. This optimal level of K is lower when δ is low and higher when δ is high, as we see from the results in Tables 4 and 5 (keeping in mind that initial leverage is higher when δ is low and lower when δ is high).

Overall, the probability of defeasance depends on the volatility of the assets σ, and the loss on defeasance depends on the fire sale discount δ. Hence we portray our results in each table along these two dimensions. We now consider how these results change when we vary the volatility of the underlying asset pool.

4.5. Varying Volatility

We now explore the resulting SIVs and risk-return tradeoffs under varying levels of asset volatility. This variation is especially interesting since the various SIVs that failed during the crisis had wide ranging levels of asset risk. In Table 6 we compare different cases for $\sigma = \{0.05, 0.10\}$, keeping $\delta = 0.05$.

Table 6. SIV outcomes when risk control K is varied. Volatility takes two values $\sigma = \{0.05, 0.10\}$ and the price of debt is fixed across all cases to the price of debt when $K = 0$, and $D_B = 0.99 D_B^*$. Under the physical measure, we have asset returns $\mu = \{0.05, 0.10\}$, respectively, for the two values of volatility.

			$\sigma = 0.05; \delta = 0.05$				
K	**0**	**1.005**	**1.015**	**1.025**	**1.035**	**1.04**	**1.045**
D_B	88.2537	88.2529	88.2511	88.2465	88.2313	88.2282	88.2175
D_B^*	89.1451	89.202	89.2154	89.4477	96.6184	96.1538	95.6938
P_B	86.4526	86.4526	86.4526	86.4526	86.4526	86.4526	86.4526
$E(L_B)$	0.0006	0.0006	0.0006	0.0005	0.0003	0.0003	0.0001
D_C	13.5474	13.5474	13.5474	13.5474	13.5474	13.5474	13.5474
$E(L_C)$	0.2154	0.2154	0.2155	0.1922	0.0294	0.0405	0.0467
PD_K	0.0000	0.0059	0.0105	0.0178	0.0298	0.0411	0.0477
PD_T	0.2873	0.2814	0.2767	0.2288	0.0000	0.0000	0.0000
PD	0.2873	0.2873	0.2873	0.2466	0.0298	0.0411	0.0477
ETD	1.0000	0.9959	0.9919	0.9821	0.7268	0.7054	0.6919
μ_B	0.0305	0.0304	0.0304	0.0303	0.0301	0.0300	0.0299
μ_C	0.3103	0.3104	0.3105	0.3231	0.3708	0.3688	0.3683
PD_K^{RN}	0.0000	0.0014	0.0027	0.0053	0.0100	0.0151	0.0183
PD_T^{RN}	0.1744	0.1730	0.1716	0.1356	0.0000	0.0000	0.0000
			$\sigma = 0.10; \delta = 0.05$				
K	**0**	**1.005**	**1.015**	**1.025**	**1.035**	**1.04**	**1.045**
D_B	78.8195	78.8127	78.8058	78.7994	78.7881	78.7785	78.7717
D_B^*	79.6156	80.0876	80.2884	80.9981	96.6184	96.1538	95.6938
P_B	77.1988	77.1987	77.1988	77.1988	77.1988	77.1988	77.1988
$E(L_B)$	0.0008	0.0007	0.0006	0.0005	0.0003	0.0002	0.0001
D_C	22.8012	22.8013	22.8012	22.8012	22.8012	22.8012	22.8012
$E(L_C)$	0.2421	0.2263	0.2263	0.2091	0.0291	0.0291	0.0343
PD_K	0.0000	0.0129	0.0152	0.0213	0.0295	0.0295	0.0349
PD_T	0.4160	0.3555	0.3532	0.3014	0.0000	0.0000	0.0000
PD	0.4160	0.3684	0.3684	0.3227	0.0295	0.0295	0.0349
ETD	1.0000	0.9929	0.9912	0.9849	0.7547	0.7547	0.7480
μ_B	0.0308	0.0306	0.0305	0.0304	0.0302	0.0300	0.0299
μ_C	0.5415	0.5505	0.5509	0.5586	0.5879	0.5885	0.5885
PD_K^{RN}	0.0000	0.0019	0.0023	0.0037	0.0057	0.0057	0.0071
PD_T^{RN}	0.2218	0.1800	0.1795	0.1436	0.0000	0.0000	0.0000

As expected, as volatility of the assets increases, allowable leverage, i.e., the maximal amount of senior notes D_B that can be issued while keeping $E(L_B) \leq L_0$, will decline. The probability of defeasance PD also increases with volatility. However, the improvement of the cases with risk controls ($K > 1$) over the case without risk controls ($K = 0$) is telling. Yield spreads decline and returns to capital notes increase, as the leverage control K implements the optimal trade off between probability of defeasance and loss on defeasance. In short, the intuitions and results support administering risk controls for such structures, and ensure improvements in the quality of liabilities irrespective of the level of asset risk.

4.6. Summary of Main Properties of the Model

We take pause here to summarize the main features and results of the model.

1. *Interaction between risk controls, fire sale discounts, and leverage*: K is bounded above by $\frac{1}{1-\delta}$, so for higher δ, the range of K that we consider in our examples is greater. Note that $D_B < A_0/K$, else the SIV will encounter defeasance immediately.
2. *When defeasance may never occur at maturity*: Recall that D_B^* is set such that $E(L_B) = L_0$, given $K = 0$. When $K > 1$, for low K, maximal debt will be such that $D_B^* < A_0/K$. As K increases, it is possible that $D_B^* = A_0/K$. When this is the case, then $A^* = H$. To see this, note that

$$A^* = A_0/D_B^* \cdot D_B = A_0/(A_0/K) \cdot D_B = D_B K = H \tag{23}$$

   So in this case defeasance can never occur at maturity, only before maturity if it occurs at all. This solidifies the intuition that as K increases, the barrier H approaches the senior note holders rollover threshold A^*.
3. *When risk controls are beneficial*: (a) Increasing K without changing the initial leverage of the SIV results in a decrease in the face value of debt to be repaid, which implied that the yield on the securities decrease (since initial $B_0 = P_B$ remains constant), particularly when the fire sale discount is small (e.g., $\delta = 0.02$). This can be seen also in the corresponding decline in percentage expected losses to senior debt ($E(L_B)$). (b) The expected percentage losses to equity (capital notes) also decline with K, but begin to rise again as K becomes increasingly stringent. Nonetheless, expected losses to capital-note holders ($E(L_C)$) are significantly lower under any $K > 1$ than when no risk control is used, i.e., $K = 0$.
4. *The intuitive trade-off in the model*: The lower loss on defeasance must to be traded off against the situation where the total probability of defeasance PD increases as K increases. Note also that in these structures the PD can be quite high, suggesting that the large-scale demise of SIVs in the financial crisis was no surprise. We have decomposed PD into the probability of defeasance at the barrier H prior to maturity (PD_K), and the probability of defeasance at maturity (PD_T) due to rollover failure. As is intuitive, when K rises, PD_K increases and PD_T falls. As noted before, if $D_B^* = A_0/K$, then $A^* = H$, so that $PD_T = 0$.
5. *Relation of best risk control and fire sale discounts*: Considering $\delta = 0.10$ (i.e., a much greater fire sale discount), the results remain qualitatively similar to those under $\delta = 0.03$. However, K must be set much higher to lower yield spreads, i.e., proxied by the level of D_B relative to P_B. Therefore, as expected fire-sale discounts rise, risk controls must be tightened (again, keeping in mind that the SIV's leverage is much higher under $\delta = 0.02$ than under $\delta = 0.10$).
6. *Risk controls are more important when volatility is higher*: We also see that the PD declines even more with risk controls (i.e., with greater K) when asset volatility is greater (e.g., when increasing σ from 0.03 to 0.05 while keeping $\delta = 0.05$).
7. *Risk controls on SIVs can be Pareto-improving*: At the right K, both $E(L_B), E(L_C)$ decline. Yield spreads also decline, and expected returns to capital note holders increase. Thus, the right level of stringency in the leverage threshold allows all investors to benefit by reducing deadweight losses otherwise borne by one (or both) of the parties.

5. Introducing Contingent Capital

In the financial crisis, many SIVs triggered defeasance at the same time, leading to large losses as fire sale discounts escalated. For example, under a fire-sale discount of $\delta = 0.10$, even with a leverage threshold of $K = 1.07$ in place, senior debt holders suffer a percentage loss of $(1 - (1 - \delta)K) = 0.037$, which is a substantial percentage loss for debt that is originally rated AAA. At the height of the crisis, some SIVs liquidated at discounts ranging from 0.30 to 0.50, which are much higher than those shown in the analyses described here. Static risk controls with large losses on defeasance offer no soft landing for senior note holders in times of financial crises, indicating that senior note holders should be further compensated for the "neglected risks" they bear (e.g., see [24]). These additional benefits may be dynamic and state-dependent, resulting in a value transfer from capital notes to senior notes in poor states of the economy. Our analysis of the weaknesses of SIVs would surely be incomplete without offering remedies.

A simple prescription for risk buffering the senior note holders is to enhance the SIV with a contingent capital clause, whereby capital note holders (or the issuer/sponsor) are required to provide additional capital to buy back a pre-specified quantity of senior notes at par (adjusted for time value of money), thus returning the leverage ratio to a safer level. This dynamic risk management approach is similar to a margin call, whereby the extra infusion/buyback is analogous to the variation margin. *Ex-ante*, this built-in capital call discourages the creation of a SIV that the issuer/sponsor knows to be too risky without sufficient upside, and promotes an equilibrium where mostly viable SIVs are created.[12] Payment of contingent capital could be ensured by credit or liquidity guarantees, much in the way that conduit sponsors of fully supported ABCP (asset backed commercial paper) guarantee payment [26]; though, the contingent capital we prescribe below would be far less than full support.

To recap, the eventualities for the SPV are binary: either asset values rise or they fall. (a) In the case when asset values rise, maintaining additional equity beyond that under the original structure results in a reduction in leverage of the vehicle, which reduces the expected return to equity holders. This was discussed in Section 2.3. Therefore, at the rollover date, equity holders withdraw just the amount of equity needed to restore the original risk-return trade-off. This feature also makes the solution stationary, as it reinitializes the SPV so that at the horizon, conditional on no defeasance, the model repeats itself; (b) In the case when asset values fall (as in the remedial case analyzed in this section), the risk of defeasance increases as does the expected deadweight loss (from fire sales), which makes it rational to inject equity in an optimal trade-off against bearing liquidation losses. An alternative solution is to defer withdrawal of equity capital on the rollover date when asset values have risen, leaving an extra buffer in place from good times that might make redundant the need to later inject contingent capital or at least reduce the size of the injection. It also acts as a capital commitment mechanism on equity holders. The model may be suitably modified to determine under what parameters either approach is more appropriate. The solution would depend, of course, on investors' risk aversion and how equity holders' other assets are correlated with those held by the SIV.

We now examine the net benefits of capital calls. Our chosen method requires that when the defeasance trigger is accessed at stopping time t, i.e., when the asset values decline to a level such that $A(t)/D_B = K$, then we have a capital infusion I provided by the capital note holders (or SIV sponsor/underwriter) to pay down D_B to some level $D'_B < D_B$ such that $A(t)/D'_B > K$. This remedy recapitalizes the SIV, replacing senior notes with capital notes without altering or liquidating the assets of the SIV.

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[12] Basel III envisages requiring contingent capital in addition to Tier 1 capital from banks. While this is not yet under consideration in the US, the Independent Commission on Banking [25] recommended that UK banks be required to hold an additional 7%–10% of contingent capital, also known as bail-in capital, where debt would automatically convert to equity on breaching a trigger level of leverage.

In effect, this capital infusion results in a partial deleveraging of the SIV, since the amount of senior debt is reduced relative to the assets in the vehicle. This remedy effectively postpones the liquidation of the SIV, and postpones or possibly eliminates the deadweight fire sale losses on defeasance, allowing time for the SIV's assets to recover. The capital note holders also stand to benefit when deadweight costs are eliminated or even delayed, but they bear the costs of this waiting period since they are the ones providing the additional capital. Thus, although the *ex-ante* expected losses to senior note holders are definitively lower when the SIV's covenants require contingent capital infusions, the question remains as to whether the expected losses to capital note holders decline as well. Another issue is that since defeasance is postponed, it is possible that defeasance at a lower barrier, H', increases losses to capital note holders even further when the vehicle is shut down. We examine these issues by extending our model.

Since fire-sale discounts make defeasance costly to one or both of the tranches in the SIV, our analysis explores whether contingent capital infusions can be Pareto-improving. When the barrier H is breached, we undertake an infusion I by capital note holders, to partially pay off senior debt at par value, adjusted for the time value of money. Therefore, new senior debt is $D'_B = D_B - I$, and the new barrier is set at $H' = D'_B K$. Implementation of this new structure on the pricing lattice requires modifying the original model, such that when asset levels are between H' and H, computations are made using D'_B instead of D_B. We also make adjustments to the probability structure on the tree to account for the fact that defeasance will now take place at $H' < H$.

Our original model equations are extended as follows:

$$D'_B = D_B - I \tag{24}$$
$$H' = D'_B K \tag{25}$$

We now implement a lattice pricing problem with two barriers, H and H'. First, we compute the price of senior debt $P'_B = B'(0)$, the expected present value of losses to senior debt $L'_B(0)$, and expected present value of losses to capital notes $L'_C(0)$, assuming that the defeasance barrier is $H' < H$ and face value of debt is $D'_B < D_B$. We denote this the "auxiliary" lattice, i.e., the lattice to which we drop if the assets dip below H and an infusion of contingent capital occurs. We then price senior debt that originates with face value D_B on a "primary" lattice, and during backward recursion, if assets $A(t) \le H$, then we replace the value of senior debt on the primary lattice with the value of debt from the auxiliary lattice *plus* the value of the infusion I that is made at that point.

The time T equations for pricing senior debt on the primary and auxiliary lattices are as follows:

$$B'(T) = \begin{cases} D'_B & \text{if } A(T) \ge A^* \\ \min[D'_B, A(T)(1-\delta)] & \text{if } H' < A(T) < A^* \\ \min[D'_B, H'(1-\delta)] & \text{if } A(T) \le H' \end{cases} \tag{26}$$

$$B(T) = \begin{cases} D_B & \text{if } A(T) \ge A^* \\ \min[D_B, A(T)(1-\delta)] & \text{if } H < A(T) < A^* \\ B'(T) & \text{if } A(T) \le H \end{cases} \tag{27}$$

Values of senior debt at all times $0 \le t < T$, are computed through the following pricing recursion:

$$B'(t) = \frac{1}{R}[q \cdot B'_u(t+h) + (1-q) \cdot B'_d(t+h)] \tag{28}$$

$$B(t) = \frac{1}{R}[q \cdot B_u(t+h) + (1-q) \cdot B_d(t+h)] \tag{29}$$

where the subscripts $\{u, d\}$ denote the up and down nodes emanating from $B'(t)$, $B(t)$. We modify the recursion equations on the auxiliary lattice to account for defeasance when $A(t) < H'$.

$$B'(t) = \min[B'(t), H'(1-\delta)], \quad \text{if } A(t) < H' \tag{30}$$

Further, we modify the recursion equations on the primary lattice to account for the fact that an amount I is paid to senior-debt holders if $A(t) < H$.

$$B(t) = B'(t) + I, \quad \text{if } A(t) < H \tag{31}$$

The expected present value of losses on senior debt is computed as follows:

$$L_B(0) = D_B e^{-rT} - B(0) \tag{32}$$

which is the difference in price between risk free debt and senior debt issued by the SIV. The expected percentage loss on senior debt is:

$$E(L_B) = \frac{L_B(0)}{D_B} \tag{33}$$

The losses on capital notes are recomputed using primary and auxiliary lattices as well, for L_C and L'_C, respectively. The equations at maturity are as follows:

$$L'_C(T) = \begin{cases} 0 & \text{if } A(T) \geq A^* \\ \min[D_C, \max[0, D_C - (A(T)(1-\delta) - D'_B)]] & \text{if } H' < A(T) < A^* \\ \min[D_C, \max[0, D_C - (H'(1-\delta) - D'_B)]] & \text{if } A(T) \leq H' \end{cases} \tag{34}$$

$$L_C(T) = \begin{cases} 0 & \text{if } A(T) \geq A^* \\ D_C - \max[0, A(T)(1-\delta) - D_B] & \text{if } H < A(T) < A^* \\ L'_C(T) & \text{if } A(T) \leq H \end{cases} \tag{35}$$

Recursion gives the expected loss value as follows, on both primary and auxiliary lattices:

$$L'_C(t) = \frac{1}{R}[q \cdot L'_{Cu}(t+h) + (1-q) \cdot L'_{Cd}(t+h)] \tag{36}$$

$$L_C(t) = \frac{1}{R}[q \cdot L_{Cu}(t+h) + (1-q) \cdot L_{Cd}(t+h)] \tag{37}$$

And if $A(t) \leq H'$, then

$$L'_C(t) = D_C - \max[0, H'(1-\delta) - D'_B / R^{T-t}] \tag{38}$$

Finally, if $A(t) \leq H$, then

$$L_C(t) = L'_C(t) + I \tag{39}$$

which leads to the expected percentage losses to capital notes

$$E(L_C) = \frac{L_C(0)}{D_C} \tag{40}$$

The expected returns on senior notes and capital notes will both also be recomputed taking into account whether or not an infusion has occurred.

Using the same parameters as in Table 4, we now consider the case whereby the required infusion is $I = 2$ to illustrate a SIV with a contingent capital clause in place. The results, which we present in Table 7, indicate that contingent capital manages deadweight losses in a Pareto-improving manner.

Table 7. SIV outcomes with infusions when risk control K is varied. The fire sale discount is fixed at $\delta = 0.05$ and the face value of debt is taken to be the same as the identical case without infusions. Compare this table (upper panel) with the results from Table 4 (included now in the lower panel). The required infusion here is $I = 2$, whereas in Table 4 no infusions are required.

$\sigma = 0.03$; $\delta = 0.05$; Infusions: $I = 2$							
K	**0**	**1.005**	**1.015**	**1.025**	**1.035**	**1.04**	**1.045**
D_B	91.8819	91.8819	91.8815	91.8801	91.8639	91.8650	91.8612
P_B	90.0210	90.0231	90.0267	90.0337	90.0399	90.0423	90.0395
$E(L_B)$	0.0005	0.0004	0.0004	0.0003	0.0001	0.0000	0.0000
D_C	9.9790	9.9769	9.9733	9.9663	9.9601	9.9577	9.9605
$E(L_C)$	0.1480	0.1482	0.1486	0.1494	0.0144	0.0212	0.0362
PD_K	0.0000	0.0001	0.0002	0.0007	0.0027	0.0041	0.0076
PD_T	0.1679	0.1678	0.1677	0.1672	0.0075	0.0101	0.0185
PD	0.1679	0.1679	0.1679	0.1679	0.0102	0.0142	0.0260
ETD	1.0000	1.0000	0.9998	0.9993	0.9464	0.9371	0.9304
μ_B	0.0301	0.0300	0.0300	0.0298	0.0296	0.0295	0.0295
μ_C	0.2984	0.2987	0.2994	0.3009	0.3384	0.3384	0.3374
PD_K^{RN}	0.0000	0.0000	0.0000	0.0001	0.0004	0.0007	0.0015
PD_T^{RN}	0.0721	0.0721	0.0719	0.0716	0.0000	0.0000	0.0000
$\sigma = 0.03$; $\delta = 0.05$; No infusions							
K	**0**	**1.005**	**1.015**	**1.025**	**1.035**	**1.04**	**1.045**
D_B	91.8819	91.8819	91.8815	91.8801	91.8639	91.865	91.8612
D_B^*	92.8100	92.8146	92.8277	92.8901	96.6184	96.1538	95.6938
P_B	90.0210	90.0210	90.0210	90.0210	90.0210	90.0210	90.0210
$E(L_B)$	0.0005	0.0005	0.0004	0.0004	0.0002	0.0002	0.0001
D_C	9.9790	9.9790	9.9790	9.9790	9.9790	9.9790	9.9790
$E(L_C)$	0.1480	0.1480	0.1480	0.1482	0.0227	0.0376	0.0604
PD_K	0.0000	0.0011	0.0034	0.0091	0.0230	0.0382	0.0623
PD_T	0.1679	0.1668	0.1645	0.1588	0.0000	0.0000	0.0000
PD	0.1679	0.1679	0.1679	0.1679	0.0230	0.0382	0.0623
ETD	1.0000	0.9988	0.9958	0.9864	0.7002	0.6614	0.6204
μ_B	0.0301	0.0301	0.0301	0.0300	0.0298	0.0297	0.0297
μ_C	0.2984	0.2984	0.2984	0.2986	0.3328	0.3293	0.3227
PD_K^{RN}	0.0000	0.0001	0.0006	0.0019	0.0062	0.0121	0.0230
PD_T^{RN}	0.0721	0.0720	0.0716	0.0702	0.0000	0.0000	0.0000

First, we examine the risk/return implications for the senior notes. Expected percentage losses here are lower (than those in the SIV without contingent capital infusions), and the price of senior notes increases correspondingly across all values of leverage risk control K, implying that credit spreads decline. The mean expected return declines as expected since the risk of these securities is lower, though only marginally. As expected, the senior notes are less risky under a structure with contingent capital infusions in place.

Next, we examine the risk/return implications for the capital notes. The expected percentage losses here remain the same (as those in the SIV without contingent capital infusions) when the leverage threshold K is low, but at higher values of K, expected losses are appreciably lower compared to the case without contingent capital infusions. The initial investment required by capital notes D_C also is lower than in the no-infusion scenario; i.e., this new structure is able to support greater leverage from the onset of the vehicle. Finally, the expected return on the capital notes for higher levels of K exceed that in the no-infusion scenario by approximately 100 bps.

Finally, we observe that the probability of defeasance is also dramatically reduced, as expected for higher levels of K, by 2 to 3 times. Overall, contingent capital infusions manage the deadweight losses of defeasance in a Pareto-improving manner. Intuitively, keeping defeasance at bay is advantageous, even for the capital note holders who are required make infusions to stave off asset liquidations.

Other Remedies

In addition to these recapitalizing approaches, we can attempt other remedies as well. For example, senior-note holders can be paid a rate of interest that is indexed to their credit rating or to that of the asset pool. As the credit rating falls, higher interest rates can be paid based on a predetermined menu, whereby the capital to make such payments come from the management fees first, and then from cash flows to capital note holders.*Ex-post*, this approach compensates the senior notes for bearing additional risk, though payment of the higher rate is contingent on the SIV's survival. More importantly, this approach discourages the SIV's issuers, ex-ante, from constructing an SIV they know to be fragile.

Alternatively, we can restore the value of the asset pool by requiring the SIV issuer/sponsor to purchase the lower rated assets that have dropped in value and to replace them with higher quality assets, thereby bringing the leverage ratio $A(t)/D_B$ back in compliance with the leverage threshold K. In effect, the capital-note holders write credit protection (i.e., a spread option) on the pool's lower rated assets. Payment is triggered when the value of the asset pool drops below the leverage threshold, and thus, also avoids or at least delays liquidation of the SIV. Once again, this ex-ante prevents knowingly fragile structured finance deal from being created.

Ultimately, in any of these remedies, "own" risk of the capital-note holders is certainly an issue. However, given that such SIVs are typically issued by deep-pocket financial firms (or too-big-to-fail banks), own risk is less of a factor. Furthermore, that these capital infusions are borne by the equity holders likens this structure to a risk-based deposit insurance premium to be borne by the bank itself, which has advantages to purely increasing capital requirements [27]; or, in the case of the SIV, equates to advantages over simply issuing a smaller senior tranche.

6. Concluding Comments

We developed a parsimonious model to analyze the design of structured finance deals and the special investment vehicles (SIVs) created to operationalize them. Our findings are as follows: First, high levels of leverage make these vehicles likely to fail when asset values drop even by a small amount, because of rollover risk. Second, the presence of fire-sale discounts, i.e., deadweight costs of defeasance, leads to fragile structures that are not designed to sustain the levels of risk inherent in the asset pool, or to ensure repayment of principal to senior note holders commensurate with a top quality credit rating. Third, losses are sensitive to risk controls, i.e., the permitted level of leverage. Fourth, risk controls vary with pool risk (i.e., volatility of the underlying assets) and expected fire sale discounts. Hence, in the presence of parameter uncertainty, designing apt risk management for such SIVs is a difficult exercise. Fifth, abandoning risk controls altogether may not be optimal for any liability holders, i.e., both senior notes and capital notes, though risk controls that are too stringent may begin to falter in their purpose. Sixth, contingent capital is clearly a design feature that is worth incorporating, but was absent in the design of SIVs prior to the financial crisis (though we note that in some cases, sponsors ex-post, willfully provided equity capital or purchased assets of their failing SIVs). The confluence of these design characteristics suggests that SIVs were not as resilient to economic shocks as they should have been, and features of the design contributed to the demise of many SIVs.

Overall, the fact that ratings and risks are highly sensitive to the fire-sale discount, pool volatility, and risk controls suggests that the senior tranche should be much smaller, and that more capital notes (i.e., a larger equity tranche) are required to sustain high-quality ratings on senior notes, as suggested in [28,29]. We recommend using contingent capital clauses that make the senior notes safer, while also improving the quality of the capital notes. Whereas contingent capital offers remediation ex-post to SIV vulnerabilities, it also likely prevents unviable SIVs from being created ex-ante, since some of the *ex-post* costs are borne by the creators of the SIV through their holdings of capital notes. When capital infusions are not possible to enforce, and contingent capital is not forthcoming, risk reduction through deleveraging the vehicle in a timely manner can also reduce expected percentage losses to both, senior debt and capital notes.

Our paper does not examine information effects.[13] Our paper argues that setting the appropriate degree of risk controls is difficult, and errors often exacerbate the effect of a drop in pool asset values; i.e., a new form of "neglected" risk (see [24]). Given the presence of neglected risks, we propose that contingent capital and deleveraging be built in to SIV covenants as important risk management policy prescriptions.

The financial crisis of 2008 was largely marked by the failure of many special investment vehicles in general, and structured investment vehicles in particular. This analysis of the pathology of the crisis and our recommendations for remediation provide guidance for the future evolution of the shadow banking sector. This paper argues that highly leveraged structures must be accompanied by clear and effective risk guidelines, and at the very least, a mandated contingent capital structure, or ex-ante deleveraging policy.

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[13] For example, pooling and tranching creates "safe" senior tranches owned by the majority of investors, leading to a dearth of informed investors in good times and resulting in insufficient risk controls, which are shown to be weak when bad times arrive [29].

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